

Unique Durable Assets

Stefano Lovo and Christophe Spaenjers*

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Abstract

This paper presents a model of trading in unique durable assets that provide idiosyncratic payoffs, such as art, luxury real estate, and firm subsidiaries. Agents make purchase and sale decisions in an auction market based on their private use value of the asset and on the expected resale revenues. Individuals with a relatively strong taste for the asset are willing to pay a high price and sell only when hit by a liquidity shock. By contrast, those deriving little pleasure from ownership act as profit-seeking speculators if acquiring the asset despite their low bids. As a result, holding periods and financial returns are negatively correlated. Furthermore, speculative activity increases in economic expansions, leading to a positive correlation between prices and voluntary sales volume. The empirical predictions of our model find support in historical art transaction data.

Keywords: auctions; durable goods; endogenous trading; private values; speculation.

JEL Codes: D44; D84; G11; G12; Z11.

*Lovo (lovo@hec.fr) and Spaenjers (spaenjers@hec.fr) are at HEC Paris, 1 rue de la Libération, 78351 Jouy-en-Josas, France. The authors thank Max Bruche, Laurent Calvet, Elroy Dimson, Thierry Foucault, Will Goetzmann, Chris Hennessy, Marcin Kacperczyk, Roman Kräussl, Di Li (discussant), Dong Lou, Andrew Metrick (discussant), Stefan Nagel, Jacques Olivier, Clemens Otto, Julien Pénasse, Raghu Rau, Avri Ravid, Luc Renneboog, Ioanid Rosu, Batchimeg Sambalaibat (discussant), Daniel Schmidt, Avaniidhar Subrahmanyam, David Thesmar, Konstantinos Zachariadis, and participants at the Yale “Art, Mind, and Markets” conference, Society for the Advancement of Economic Theory 2014 (Tokyo), European Finance Association 2014 (Lugano), Research in Behavioural Finance Conference 2014 (Rotterdam), Conference on Financial Economics and Accounting 2014 (Atlanta), and seminars at Cambridge Judge Business School, Cass Business School, HEC Paris, London School of Economics, Luxembourg School of Finance, Queen Mary University of London, Tilburg University, and University of Southampton for valuable comments. We gratefully acknowledge financial support from Labex Ecodec. All errors are ours.

1 Introduction

A Francis Bacon triptych, an estate in Palm Beach, and an airline subsidiary are unique and durable goods for which the level of enjoyment—the pleasure and pride of possession, or the industrial synergies with the parent company—varies with the identity of the owner. Such assets generally trade infrequently and through owner-initiated auctions or auction-like mechanisms. How much a bidder is willing to pay will depend on his expected private use value as an owner. At the same time, however, the possibility to resell the asset introduces a common-value element that is related to the *distribution* of private use values in the population.

The goal of our paper is to develop a theory of rational trading in unique durable assets that provide non-tradable idiosyncratic payoffs. We present a model that is informed by the institutional mechanics of the art auction market, but it can be applied to other markets in which such goods are being traded infrequently and endogenously. Our model is simple and stylized, and yet it produces a rich set of empirical implications that find support in art transaction data.

We consider an infinite-horizon economy in which individuals buy and sell artworks through English auctions. An individual acquires an artwork if he is the highest bidder and is willing to pay at least the reserve price. He then owns the artwork and enjoys an “emotional dividend”—an individual-specific non-tradable payoff or use value—in each period, until forced to sell by a liquidity shock (because of death, debt, or divorce) or until he voluntarily decides to put the item up for sale (because the expected revenues from auctioning the artwork exceed the value of holding on to it). Voluntary consignments to auctions are accompanied by a reserve price chosen by the art owner, but both successful transactions and unsuccessful sale attempts (*i.e.*, “buy-ins”) imply certain transaction costs.

We assume that the population of bidders is renewed in every period, and that bidders’ tastes are i.i.d. Each individual knows his own taste—or “type”—and the distribution from which private tastes are drawn, but cannot observe other individuals’ tastes. In each period, the

exogenous fundamentals in our economy consist of the probability that the current art owner is hit by a liquidity shock, the number of bidders in case of an auction, and a parameter that maps bidders' tastes into emotional dividends, *i.e.*, into the monetary equivalents of the pleasure of owning the artwork during one period. These fundamentals can be affected by the state of the macroeconomy, which follows a Markov process.

We show that our economy has a unique symmetric truthful equilibrium, which is identified by an appropriate state-contingent bidding function. Crucially, each bidder's valuation is endogenous to the distribution of tastes in the population, as it consists not only of a non-financial component (*i.e.*, the expected present value of the stream of emotional dividends until resale), but also of a financial one (*i.e.*, the expected present value of the revenues upon resale). The equilibrium also describes how an owner's decision whether to sell (and his reservation price) depend on his own type, the distribution of tastes in the population, and the macroeconomic state. An individual's optimal strategy as an owner affects his expected holding period, which in turn determines the relative importance of the financial component in his valuation as a bidder.

In the benchmark case of a stationary economy without macroeconomic cycles, each owner behaves either as a "collector" who sells the artwork only if forced to do so, or as a "speculator" who shortly after his purchase consigns the artwork to auction with a relatively high reserve price. The strategy of an owner depends both on his own type and on the distribution of types in the population: there exists an endogenous threshold taste level above (resp. below) which an owner is a collector (resp. speculator). It follows that individuals who expect to derive a lot of pleasure from ownership bid aggressively, and behave like collectors if they end up owning the artwork. Because they only sell when hit by a liquidity shock, they are not able to set a positive reserve price upon resale, and hence realize low proceeds on average. Although collectors have the highest level of equilibrium utility, they realize the lowest financial returns if forced to resell. By contrast, individuals with relatively low emotional dividends behave like speculators if winning the auction despite their low bids. Their aim is to resell relatively quickly, and their

financial return will on average be above that realized by collectors, because they pay less at purchase and can set a relatively high reserve price for most resales. A numerical solution of our model illustrates these equilibrium properties.

We then derive a number of comparative statics for our benchmark model. An increase in the number of potential buyers, a decrease in the probability of a liquidity shock, and a reduction in auction transaction costs all increase the expected revenues upon resale, and therefore imply higher bids and prices. These changes also translate into an increase of the threshold emotional dividend level that determines equilibrium behavior, so that a fraction of the existing collectors find it optimal to be speculators instead.

Next, we move from considering a stationary economy to a world with business cycles. We assume that the number of bidders and the magnitude of the emotional dividends derived from art ownership are lower in recessions than in expansions, while the probability of a liquidity shock is higher. We show that the equilibrium fraction of speculators—in other words, voluntary sellers—among owners is higher in expansions. We then simulate a trading history for a set of artworks using our numerical solution. Aggregated across all periods, we find that the relative frequency of speculative sales decreases with the holding period, and that average gross returns and holding periods are negatively correlated, in line with expectations. That speculative trades are observed more quickly than other (re)sales implies that repeat-sales art price indexes in our economy have to be adjusted downwards as more transaction information becomes available. The simulations also show that owners' valuations, transaction prices, and voluntary trading volume follow the state of the economy. Importantly, however, transaction-based price indexes overestimate the true sensitivity of art values to economic cycles.

We then explore how differences in the distribution of tastes in the population affect equilibrium outcomes. More specifically, we define “masterpieces” as artworks from which most individuals derive higher emotional dividends. We find that the market for masterpieces is characterized not only by higher prices but also by less speculation and therefore by longer average

holding periods than the market for “ordinary” artworks.

In our empirical part, we consider some of the main implications of the model. One key prediction is that holding periods and annualized rates of price appreciation between purchase and sale should be negatively correlated, as speculators hold for short periods and realize the highest capital gains. We find strong evidence in support of this prediction in a database of repeated sales of art at auctions worldwide between 1982 and 2007. Next, our model also predicts that art prices should be positively affected by permanent increases in the population of bidders, that economic expansions are associated with art market booms, and that higher resale values may induce some art owners to sell voluntarily, which is consistent with observed art market cycles. Finally, we find longer holding periods for masterpieces, again in line with our model’s predictions.

At the end of this paper, we provide a discussion of possible extensions of our model. We argue that introducing a private market next to the auction market, endogenizing the entry and exit of bidders, or allowing for the existence of substitutes or for temporal variation in tastes would not qualitatively change the insights from our model. Yet, relaxing the assumption that the distribution of tastes is common knowledge introduces the possibility of bubble-like price patterns. As such, a simple extension of our model could explain why prices of contemporary art are more volatile than those of Old Masters.

1.1 Related Literature

To our knowledge, this paper is the first to model the equilibrium behavior and outcomes in an auction market for durable goods that provide idiosyncratic payoffs. By doing so, it contributes to a number of strands in the literature.

First, several papers have examined how resale opportunities introduce a common-value element and affect optimal bidder behavior at auctions (e.g., Bikhchandani and Huang (1989),

Gupta and Lebrun (1999), Zheng (2002), Haile (2001, 2003), Gârleanu and Pedersen (2003), Garratt and Tröger (2006), Hafalir and Krishna (2008)).¹ These studies generally model how a good is allocated to its end user after an initial auction and a short period of post-auction trading among potential users, and compare welfare and revenues for different auction mechanisms. By contrast, our paper models the trade in assets that outlive current market participants, and focuses on how the distribution of private values and macroeconomic conditions determine valuations, owners’ strategies, and investment returns.² We see our perspective as a necessary complement to the current literature.³

Second, a number of papers have explored the relation between non-financial benefits and average financial returns for investments such as private equity and SRI (e.g., Moskowitz and Vissing-Jørgensen (2002), Bollen (2007), Renneboog, Ter Horst, and Zhang (2011)). Other studies focus specifically on the role of private values in determining returns in collectibles markets. Goetzmann and Spiegel (1995) present a simple model in which investors pay for art according to their private values. This generates a winner’s curse over short holding periods, while over longer intervals the sign of the “private value return” depends on the evolution of the distribution of private values. Mandel (2009) calibrates an asset pricing model in which a conspicuous consumption utility dividend endogenously lowers the returns on art investments.

¹Other papers study the role of resale possibilities in competitive or “search” markets. Rust (1985) presents a model of trading in durable assets that deteriorate over time. Bayer, Geissler, and Roberts (2013) identify different types of flipping behavior in the housing market. House and Ozdenoren (2008) use a matching model to show that resale concerns create a tendency for buyers of durable goods to conform to the average preference in their product choice. Gavazza (2011) examines how allocations and prices in markets for real assets such as aircraft are affected by the thickness of the asset market.

²Our model differs from the above-mentioned papers in at least three crucial aspects. First, we consider a (potentially non-stationary) infinite-horizon economy in which ownership is necessarily temporary. This allows us to study variation in (endogenous) holding periods and returns between investor types, and to relate both prices and volume to business cycles. Most existing papers focus on two-period set-ups in which an initial auction is followed by a take-it-or-leave-it offer or another auction. An exception is Gârleanu and Pedersen (2003), who consider the case of repeated endogenous trading over finitely many periods. However, their focus is on the effect of the information environment and the trading mechanism on auction outcomes. Second, in our model the population of bidders is constantly renewed. Third, we take into account the existence of reserve prices and transaction costs, allowing us to model owners’ strategies in a realistic fashion.

³Biais and Bossaerts (1998) consider the trading of a finitely-lived asset until its maturity, and analyze how departures from a common-prior assumption affect price behavior.

Jovanovic (2013) and Dimson, Rousseau, and Spaenjers (2014) investigate whether the types of wines that generate higher non-financial dividends deliver lower financial returns. Yet, these papers do not directly model the equilibrium behavior of market participants who differ in their private use values.⁴

Third, there is an empirical literature that examines endogenous sale decisions and reserve prices in the markets for illiquid assets, together with their effects on the estimation of returns. Goetzmann and Peng (2006) recognize that reserve prices in housing sales must be a function of both private values and expectations of resale revenues, and argue that observed transaction prices may therefore not accurately reflect “market values”. Cochrane (2005) shows how accounting for the success bias in observed valuations of venture capital projects dramatically lowers estimates of performance. Korteweg and Sorensen (2010, 2013) address the endogeneity of valuations and trading in the venture capital and real estate market using a new dynamic sample selection model; their procedure for estimating returns has recently been applied to the art market by Korteweg, Kräussl, and Verwijmeren (2013). Our paper adds to this work by showing how endogenous trading can create a negative correlation between holding periods and returns, and may lead to “revision problems” (e.g., Korteweg and Sorensen (2013)) with repeat-sales price indexes and to an overestimation of the covariance of durable asset prices with business cycles.

The remainder of this paper is organized as follows. The next section presents the model, defines the equilibrium concept, and shows existence and uniqueness of the equilibrium. Section 3 describes the equilibrium properties in a stationary economy, while Section 4 examines the effects of business cycles. Section 5 analyzes how differences in the distribution of tastes affect the equilibrium. Section 6 empirically tests some key predictions of our model. Section 7 discusses possible extensions, and Section 8 concludes. Proofs can be found in the Appendix.

⁴Also related in spirit to our work is recent research on how expected returns on financial assets are driven by the distribution of investors’ liquidity preferences (e.g., Shen and Yan (2014)).

2 The Model

We consider an infinite-horizon economy in which risk-neutral individuals exchange and enjoy an artwork. Time is discrete and the discount factor equals $\delta \in (0, 1)$. We denote the state of the economy at time t by $\omega_t \in \Omega$, where Ω is finite. We assume that ω_t evolves according to a stationary Markov process, *i.e.*, $\Pr(\omega_{t+1} = \omega' | \omega_t = \omega) = p_\omega(\omega')$, for all t and any $\omega, \omega' \in \Omega$. At each time t , the state of the economy is common knowledge.

In each period of ownership of the artwork, an individual i receives a non-tradable and non-financial “emotional dividend” $\rho_{\omega_t} e_i$. One can interpret this value as the maximum amount of money that the individual would be willing (and is able) to pay for enjoying the item during period t .⁵ The magnitude of the emotional dividend is individual-specific, as it is a function of a personal taste parameter $e_i \in [0, 1]$, which we refer to as the “type” of individual i . All individuals know their own type but cannot observe other people’s types. Next to tastes, also the state of the economy affects the levels of the emotional dividends: ρ_{ω_t} is a positive state-dependent constant. For example, changes in wealth can have an impact on the willingness to pay for luxury consumption (Aït-Sahalia, Parker, and Yogo (2004), Hiraki et al. (2009)).

In every period t following the acquisition of the artwork, there is an exogenous probability d_{ω_t} that the art owner faces a liquidity shock associated with death, debt, or divorce.⁶ The probability of a liquidity shock is the same for all owners, but might vary over time with macroeconomic conditions, *i.e.*, with the state of the economy ω_t . At the beginning of each period, an owner will face one of two situations. Either he is hit by a liquidity shock, and then he is forced to sell with a reserve price r equal to zero. Or he is not hit by a liquidity

⁵Although the emotional dividend can be thought of as a kind of “rental equivalent”, it is important to stress that we assume non-tradability of the dividend. In other words, ownership is a condition for enjoyment. The observation that there does not exist a well-developed art rental market is consistent with the hypothesis that that the pride of possession is more important than viewing pleasure per se (Frey and Eichenberger, 1995).

⁶The CEO of auction house Christie’s puts it as follows: “Our market has always been driven by the three D’s: death, debt, and divorce. No matter what the state of the market, divorce happens, people die, and debt has to be paid” (Newsweek (2010)).

shock, and then he can either keep the artwork for another period or consign the artwork to an auction house with an optimally selected reserve price r . If the work comes up for auction, the auctioneer runs an English auction. Let b denote the highest bid. If $b \geq r$, then the artwork is sold: the winning bidder pays b , the seller receives $(1 - \gamma_1)b$, and the auctioneer is paid a fee of $\gamma_1 b$. If $b < r$, then the artwork is “bought in”: the seller keeps the artwork, but has to pay a fee of $\gamma_0 r$ to the auctioneer as a compensation for running the auction.

The population of bidders changes over time with new individuals entering and old individuals leaving the art market. We model this process in the simplest possible way: we assume that the population of potential buyers is renewed in every auction. Formally, if the artwork is auctioned at time t , there are n_{ω_t} bidders. Bidders’ types, *i.e.*, tastes, are i.i.d. on the interval $[0, 1]$ drawn from a common-knowledge c.d.f. F and density function f . The winner of the auction becomes the next owner of the artwork. All the other bidders exit the market. Thus the auction presents each bidder with his unique opportunity to acquire the artwork and enjoy it until resale. Bidders do not know the reserve price or the reason why the current owner is selling.⁷ Let h^t denote the information that is publicly available at time t . This information includes the current and past states of the economy, and the dates and outcomes of past auctions. Let H be the set of possible histories. To simplify notation, we will denote by \Pr_t and E_t the conditional probability and expectation operator given $h^t \in H$.

2.1 Equilibrium Concept

We focus on symmetric and truthful equilibria. Symmetry means that at any given time t potential buyers of the same type e will bid identically. Truthfulness means that each auction participant uses the weakly undominated strategy of increasing his bid until he reaches his

⁷Given the private-value and English-auction framework, such knowledge would not affect bidders’ behavior.

valuation of the artwork, at which point he stops bidding.⁸ This implies that an item will be bought in only if the highest bidder's valuation is below the reserve price. In the case of a successful sale, the price paid by the winning bidder equals the maximum of the reserve price and the highest of his competitors' valuations.

We can then compute the expected cashflow from a sale as follows. Denote by $b_t(e)$ the value put on the artwork by a bidder of type e participating in the auction taking place at time t . Furthermore, suppose that $b_t(\cdot)$ is a non-decreasing function, and let $\tilde{e}_t^{(1)}$ and $\tilde{e}_t^{(2)}$ denote the highest and second-highest type among time t bidders, respectively. The owner's expected cashflow from an auction at time t with a reserve price of r is equal to:

$$R_t(r) = \Pr_t[b_t(\tilde{e}_t^{(1)}) \geq r](1 - \gamma_1)E_t[\max\{b_t(\tilde{e}_t^{(2)}), r\} | b_t(\tilde{e}_t^{(1)}) \geq r] - \Pr_t[b_t(\tilde{e}_t^{(1)}) < r]\gamma_0 r \quad (1)$$

The first term on the r.h.s. of (1) is the probability that the highest bidder is willing to pay at least r times the expected revenue from the auction, net of transaction costs, in the case of a successful sale. The second term is the probability that no bidder is willing to pay at least r times the payment to the auctioneer in the case of a buy-in.

How much will a potential buyer of type e bid in an auction taking place at time t ? If he wins the auction, he enjoys $\rho_{\omega_t}e$ in period t . At the start of the next period, either he is hit by a liquidity shock, and then he must auction the artwork but cannot set a positive reserve price, so his payoff would be $R_{t+1}(0)$. Or he is not forced to sell, and then his continuation payoff equals $V_{t+1}(e)$, with $V : H \times [0, 1] \rightarrow \mathbb{R}$, denoting the time $t + 1$ value function of an owner of type e not hit by a liquidity shock. Hence, we have that:

$$b_t(e) = \rho_{\omega_t}e + \delta E_t[d_{t+1}R_{t+1}(0) + (1 - d_{t+1})V_{t+1}(e)] \quad (2)$$

⁸In other words, we focus on equilibria in the ascending (or Japanese) format of the English auction, in which the price continuously increases as long as there are at least two bidders willing to pay this price.

We are now ready to pin down the equilibrium value of $V_t(e)$. We know that an owner not hit by a liquidity shock at time t has to choose between keeping the artwork for at least one more period or deciding on a reserve price and auctioning the artwork. Hence, $V_t(e)$ must satisfy the following functional equation:

$$V_t(e) = \max\left\{ \begin{aligned} &\rho_{\omega_t}e + \delta E_t[d_{t+1}R_{t+1}(0) + (1 - d_{t+1})V_{t+1}(e)], \\ &\max_r \left[R_t(r) + \Pr_t[b_t(\tilde{e}^{(1)}) < r] \left(\rho_{\omega_t}e + \delta E_t[d_{t+1}R_{t+1}(0) + (1 - d_{t+1})V_{t+1}(e)] \right) \right] \end{aligned} \right\} \quad (3)$$

The first term on the r.h.s. of (3) is the payoff from keeping the artwork. It consists of the sum of (i) the emotional dividend in period t and (ii) the present value of the weighted average of the expected resale revenue in case of a liquidity shock in the next period and the expected continuation value in the absence of a liquidity shock. The second term on the r.h.s. of (3) is the expected payoff from auctioning the artwork at time t . It consists of the sum, given the optimally chosen r , of (i) the expected cashflow from the auction and (ii) the probability of a buy-in times the expected payoff from keeping the artwork. From expression (2), equation (3) can be rewritten as follows:

$$V_t(e) = \max\{b_t(e), \max_r [R_t(r) + \Pr_t[b_t(\tilde{e}^{(1)}) < r]b_t(e)]\} \quad (4)$$

We can see from equations (2) and (4) that the willingness to pay of each bidder is the sum of the expected present value of the stream of emotional dividends until resale and the expected present value of the revenues upon resale. Note that once the bidding function $b : H \times [0, 1] \rightarrow \mathbb{R}$ is given, the functions $R : H \times \mathbb{R} \rightarrow \mathbb{R}$ and $V : H \times [0, 1] \rightarrow \mathbb{R}$ —which describe each type's decision whether to sell and the optimal reservation price—result from equations (1) and (4). Hence an equilibrium is identified by an appropriate bidding function:

Definition 1 *A symmetric truthful equilibrium is characterized by a bidding function $b : H \times$*

$[0, 1] \rightarrow \mathbb{R}$ such that in every period t and after every history $h^t \in H$:

1. The expected revenue from an auction with reserve r is the $R_t(r)$ satisfying equation (1).
2. If an auction takes place, a type e bidder bids up to the $b_t(e)$ satisfying equation (2).
3. A type e owner's value function $V_t(e)$ satisfies equation (4).

A symmetric truthful equilibrium is Markov (henceforth STM equilibrium) if every bidder's highest bid at time t only depends on his type e and on the state of the economy ω_t . In this case, $b : \Omega \times [0, 1] \rightarrow \mathbb{R}$, and b is a state-contingent bidding function.

Note that in any equilibrium the value function $V_t(\cdot)$ is increasing in e . An owner of type e can always secure the same expected future resale revenue as an owner of type $e' < e$ by adopting that type's resale strategy and reserve price. In addition, he will enjoy an extra emotional dividend $e - e' > 0$ in each period of ownership. Equation (2) then implies that the bidding function is strictly increasing in e . We can now state the following result:

Proposition 1 *There exist a unique state-contingent bidding function $b : \Omega \times [0, 1] \rightarrow \mathbb{R}$ that forms an STM equilibrium. In any state ω , a bidder's equilibrium bid is continuously increasing in his type.*

To prove the proposition, we consider the mapping from future state-contingent bidding functions into the current state-contingent bidding function. We then show that this mapping is a contraction and hence has a unique fixed point that is the STM equilibrium of the economy. This also implies that recursively applying this mapping to any arbitrarily chosen state-contingent non-decreasing bidding function results in a convergence to the STM equilibrium. We will later use this contraction property to numerically determine b for different parameter values.

3 Stationary Economy

3.1 Equilibrium Properties

We first consider the properties of the equilibrium in a market unaffected by macroeconomic shocks, *i.e.*, $\rho_\omega = 1$, $d_\omega = d$, and $n_\omega = n$ for all states $\omega \in \Omega$. In a stationary equilibrium, the expected revenue from selling the artwork at any time t only depends on the reserve price set by the seller. Hence in a symmetric truthful equilibrium we can drop all time subscripts in equations (1)–(4), and rewrite equation (4) as follows:

$$V(e) = \max\{b(e), \max_r [R(r) + \Pr[b(\tilde{e}^{(1)}) < r]b(e)]\} \quad (5)$$

Equation (5) implies that, in the absence of a liquidity shock, the decision whether to sell will depend solely on the type e of the owner. Two kinds of behavior therefore emerge in equilibrium, namely that of a “collector” who only sells when hit by a liquidity shock, and that of a “speculator” who sells at the earliest opportunity with an optimal reserve price. Whether an owner behaves like a collector or a speculator is endogenously determined.

When a liquidity shock occurs, the seller’s reserve price is zero. It is therefore possible that an artwork is bought by a bidder whose emotional dividend is below the one of the seller. Denote by $G_t(e)$ the probability that at some time t the artwork is owned by an individual whose type is below e . To the equilibrium play corresponds a transition operator \mathcal{P} mapping any given c.d.f. G_t into G_{t+1} . We denote by $G^* : [0, 1] \rightarrow [0, 1]$ the fixed point of \mathcal{P} , *i.e.*, $G^* = \mathcal{P}G^*$. Thus, G^* is the stable cumulative distribution of owner types that is associated with the equilibrium.

Consider an individual of type e in the STM equilibrium. Let $P_0(e)$ denote what this individual expects to pay when buying the artwork at auction. We denote by $\tau(e)$ his expected holding period, by $r(e)$ his optimal reserve price when voluntarily selling the artwork, and by $U(e)$ his ex-ante expected overall payoff. All these expectations are conditional on the distri-

bution of owners being G^* and on individuals' equilibrium strategies. The following result then summarizes the properties of our equilibrium:

Proposition 2 *In a stationary economy, the STM equilibrium satisfies the following properties:*

1. *There exists an $e^* \in [0, 1]$, such that owners of type $e \geq e^*$ behave like collectors, whereas owners of type $e < e^*$ behave like speculators.*
2. *The expected price $P_0(e)$ paid by a winning bidder of type e is increasing in e .*
3. *The reserve price $r(e)$ set by a speculator of type e when voluntarily selling the artwork is non-decreasing in e . Sales resulting from liquidity shocks take place at a reserve price of zero, but the winning bid will never be below $b(0) > 0$.*
4. *The expected holding period $\tau(e)$ is weakly increasing in e . It equals $1/d$ for collectors.*
5. *A bidder's equilibrium expected utility $U(e)$ is an increasing concave function of e .*
6. *The long-term distribution of owner types is $G^* = F^n$.*

Proposition 2 implies that individuals with relatively high emotional dividends behave like collectors, whereas individuals with relatively low emotional dividends behave like speculators. The threshold e^* corresponds to the emotional dividend that makes an owner indifferent between selling immediately (with an optimally chosen reserve price) or selling only once hit by a liquidity shock (without a reserve). The costliness of auctions guarantees that collectors do not sell voluntarily.⁹ On average, collectors will pay higher prices than speculators. All sales by collectors come without a reserve, while speculators set optimal reserve prices when they are not hit by liquidity shocks. It follows that the cashflow conditional on the actual sale of the artwork is higher for speculators. Moreover, in voluntary sales, both the probability of a buy-in and the expected resale price are positively correlated with the price paid by the speculator at entry, as

⁹An owner only wants to sell to a buyer who likes the artwork much more than himself, as the buyer's willingness to pay must exceed the sum of the seller's valuation and the costs associated with a successful transaction. When the owner's type e is relatively high, the probability of finding such a buyer is small. Moreover, there would be a large cost $\gamma_0 r$ each time the work is bought in.

the reserve price set by the speculator is an increasing function of his type e . Because collectors sell only if forced by a liquidity shock, they hold for a relatively long period on average. Moreover, even though they put in the highest bids, they can expect the highest utility levels, because they still pay a price below their own valuation and are more likely to win the auction than speculators.

The properties of the equilibrium imply that speculators can expect much higher financial returns upon resale than collectors. The existence of transaction costs makes it necessary to distinguish between gross and net returns. Consider an individual who buys the asset at t for a price of P_t , then may experience one or more buy-ins, and eventually resells successfully at date t' for $P_{t'}$. The gross return per period can then be computed as $\left(\frac{P_{t'}}{P_t}\right)^{\frac{1}{t'-t}}$. We define the net return as the internal rate of return taking into account all the fees that go to the auctioneer for the sale at time t' and for any unsuccessful auctions between t and t' . We will denote by $\pi_G(e)$ and $\pi(e) \leq \pi_G(e)$ the expected gross and net return per period for an individual of type e .

3.2 Numerical Solution

In this section we present a numerical solution of our model to further illustrate our findings. We set $n = 10$, $\gamma_1 = 25\%$, $\gamma_0 = 5\%$, $d = 0.1$, and use a discount rate δ of $1/1.05$.¹⁰ Next, we assume that, for an ordinary artwork, tastes e are distributed in such a way over the interval $[0, 1]$ that a majority of bidders would derive a relatively low amount of pleasure from the artwork, but some individuals may enjoy the artwork much more. More specifically, we assume that bidder types follow a beta distribution with parameters $\alpha = 0.5$ and $\beta = 2$. Figure 1 shows the corresponding c.d.f. and density function.

Figure 2 shows our numerical solution given these parameter values. Panel (a) presents the

¹⁰In today's art auction markets, sellers get the hammer price minus a seller's commission, while buyers pay the hammer price plus a buyer's premium. Both can of course be considered as a tax on the seller, as we do here. Taken together, the commission and the premium typically amount to about 25%, although the cost can be higher for low-value items. The fraction of the reserve price that potential sellers have to pay to the auction house in case of a buy-in varies across sales, but it lies substantially below the cost of a successful sale.

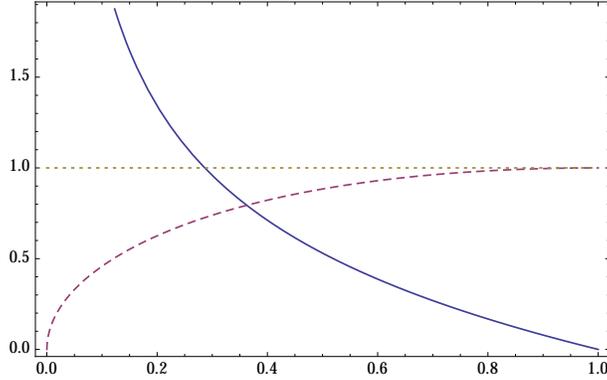
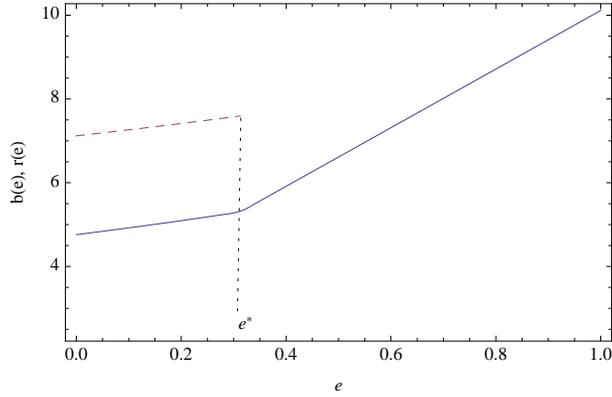


Figure 1: This figure shows the c.d.f. (dashed line) and density function (solid line) of types e under the assumption that e follows a beta distribution with parameters $\alpha = 0.5$ and $\beta = 2$.

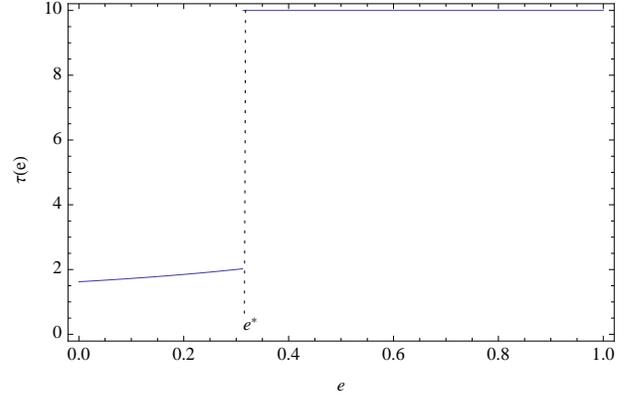
bidder's value function as a function of e . Speculators are the owners with an $e < e^* = 0.31$, while for collectors $e \geq e^*$. The panel also shows the reserve price $r(e)$ associated with voluntary sales by speculators. Note that $r(0) > b(e^*)$, which means that speculators will not voluntarily sell to another speculator. This also implies that, conditional on the exit auction of a speculator being voluntary and successful, the average ratio of the sale price over the purchase price is large. In our numerical solution, it equals 1.49 for the speculator with an e just below e^* . However, considering the magnitude of transaction costs and the fact that also speculators may be hit by a liquidity shock, it is no surprise that the expected net financial return per period is substantially lower.

Panel (b) shows the expected holding period as a function of e . For collectors, it is equal to $1/d = 10$ periods. For speculators, the expected holding period is much lower, although it increases with e , because agents who enjoy a higher emotional dividend set higher reserve prices.

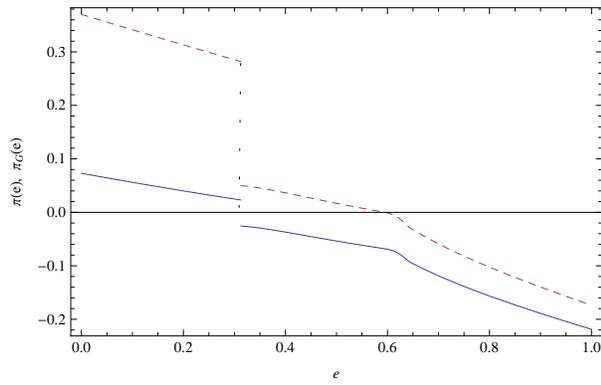
Panel (c) shows how the expected gross and net financial return per period vary with e . Financial returns are decreasing in e mainly because the expected entry price $P_0(e)$ is increasing in e , but also because the expected resale revenues are $R(0)$ for all $e \geq e^*$. Nevertheless, a collector's expected gross return (-3.1% on average) is positive as long as his type is not too high. After taking into account transaction costs, the average $\pi(e)$ for collectors is -9.5% . The kink in returns at e close to 0.6 reflects the fact that only collectors with a sufficiently large



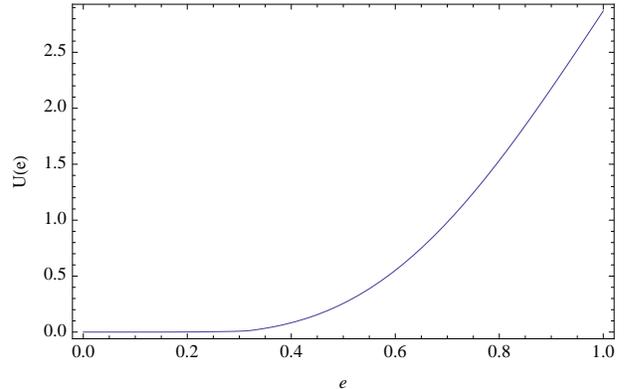
(a) Bidders' valuations and speculators' reserve prices



(b) Expected holding period



(c) Expected gross and net financial return



(d) Expected equilibrium utility

Figure 2: Panel (a) represents the bidder's value function (solid line) and the optimal reserve price (dashed line) as functions of e . Panel (b) presents the expected holding period as a function of e . Panel (c) presents the expected gross (dashed line) and net (solid line) financial return per period as functions of e . Panel (d) presents the expected equilibrium utility as a function of e .

e are willing to buy at the high reserve prices set by speculators. Speculators' expected gross returns (29.9% on average) are substantially above those of collectors. Also their expected net returns are positive (3.2% on average). The discontinuity in the financial returns at e^* is due to the discontinuity in expected holding periods and expected resale revenues around e^* .

Panel (d) shows that a bidder's ex-ante expected equilibrium utility is continuously increasing in his type e . So even though bidders with a very high e expect the lowest financial returns if they need to resell, they have the highest ex-ante expected equilibrium payoff taking into account the magnitude of their emotional dividends and the likelihood of buying the artwork.

Table 1 summarizes the results of our numerical solution. Given that speculators have much

Table 1: Distribution of tastes, bidding, ownership, holding periods, and financial returns

e^*	0.31
Fraction of speculators among bidders	75.1%
Fraction of speculators among owners	5.7%
Average holding period for speculators	1.92
Average holding period for collectors	10.00
Average holding period for all owners	9.54
Average financial gross return per period for speculators	29.9%
Average financial gross return per period for collectors	-3.1%
Average financial net return per period for speculators	3.2%
Average financial net return per period for collectors	-9.5%

shorter average holding periods and much higher average gross returns than collectors, there will be a negative correlation between holding periods and hammer price appreciation rates in our economy. In the next section, we will illustrate this result using numerical simulations.

3.3 Comparative Statics

We can describe how an equilibrium in our economy is affected by permanent shocks to the number of potential bidders per period n , the probability of a liquidity shock d , the discount factor δ , or the magnitude of the transaction costs:

Corollary 1 *Increases in n or δ , and decreases in d , γ_0 , or γ_1 have the effect of increasing $V(\cdot)$, $b(\cdot)$, and $R(\cdot)$. Increases in n , and decreases in d , γ_0 , or γ_1 , also have the effect of increasing e^* .*

An increase in the number of bidders n increases the expected proceeds from selling the artwork for all reservation price levels. When δ is higher, individuals put a higher value on future emotional dividends and cashflows. A lower d implies that owners are less likely to be hit by a liquidity shock that forces them to sell at a zero reserve price. A decrease in γ_0 or γ_1 leads to a lower fee payable by owners in the case of a buy-in or a successful auction, respectively. All these factors increase the value of ownership, translating into larger bids and auction proceeds.

Interestingly, a reduction in transaction costs has a multiplicative effect on sellers' net proceeds, as it raises both each auction participant's valuation for the artwork and the fraction of the winning bid that goes to the seller. The effect on the willingness to pay of bidders is absent in static auction settings without resale possibilities.

The intuition behind the increase in e^* after an increase in n , a decrease in d , or a decrease in the transaction cost rates is as follows. An owner of type e^* is indifferent between keeping the asset until hit by a liquidity shock and selling immediately at an optimal reserve price. If the number of bidders increases, the expected resale revenues go up more than the value of ownership, and therefore this owner will strictly prefer auctioning the artwork. In other words, the threshold e^* that separates speculators from collectors goes up. In moving from the old stationary equilibrium to the new one, some collectors will thus become speculators, temporarily raising the volume of auctions. The effect of a decrease in the probability of a liquidity shock or of the transaction costs on e^* is similar.¹¹ However, the immediate effects on trading volume are in principle ambiguous. A lower liquidity risk increases the number of voluntary sales, but also reduces the number of forced sales. Lower transaction costs increase the number of voluntary consignments but also the reserve prices set by speculators and therefore the probability of a buy-in. Indeed, our numerical solution shows that trading volume goes down when transaction costs decrease.

4 Non-Stationary Economy

4.1 Numerical Solution

We now consider a non-stationary economy with macroeconomic cycles. We numerically solve a model in which there are two possible economic states: recession ($\underline{\omega}$) and expansion ($\bar{\omega}$).

¹¹The effect on e^* of an increase in δ is unclear. The larger expected auction revenues give an incentive to sell immediately, but a larger δ also implies a higher value of delaying the sale by another period.

Table 2: Recessions and expansions

	$\underline{\omega}$	$\bar{\omega}$
e_{ω}^*	0.18	0.34
Fraction of speculators among bidders	60.0%	77.6%
Fraction of speculators among owners	1.7%	4.8%

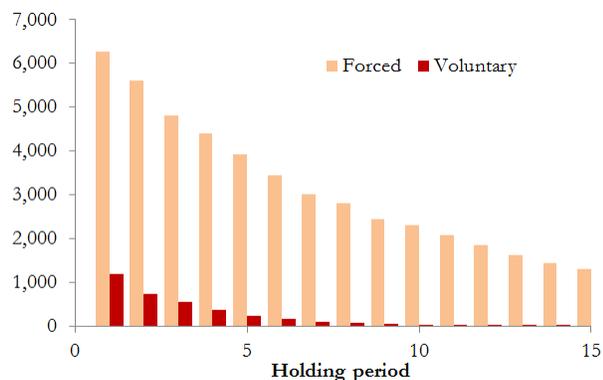
In economic busts, keeping tastes fixed, the monetary equivalent of the pleasure of enjoying art during one period is lower, *i.e.*, emotional dividends are lower. Moreover, recessions are associated with a lower number of bidders and a higher probability of a liquidity shock.

We numerically solve the model for the following parameter values: $\gamma_1 = 25\%$, $\gamma_0 = 5\%$, $d = 0.1$, $\delta = 1/1.05$, $\rho_{\underline{\omega}} = 0.5$, $\rho_{\bar{\omega}} = 1$, $n_{\underline{\omega}} = 8$, $n_{\bar{\omega}} = 12$, $d_{\underline{\omega}} = 0.11$, and $d_{\bar{\omega}} = 0.09$. For F , we use the same beta distribution as before. State-transition probabilities $p_{\omega}(\omega)$ equal 0.75 for all $\omega \in \{\underline{\omega}, \bar{\omega}\}$.

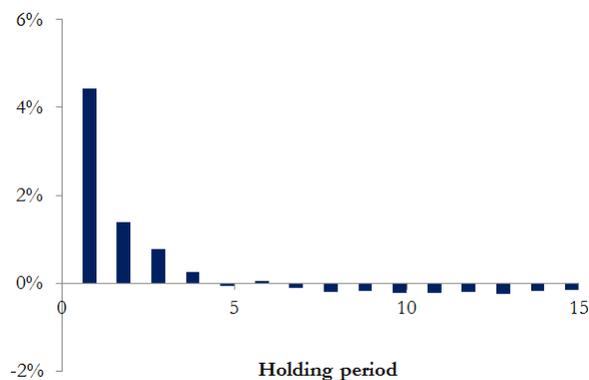
Because emotional dividends and resale values are higher in expansions than in recessions, bids, owners' values, and auction revenues will also be positively impacted. Furthermore, the threshold level of tastes separating collectors from speculators is higher in economic booms, implying that there are types who behave like a collector in recessions, but like a speculator in expansions. The stable equilibrium fraction of speculators among bidders is therefore higher in expansions, as shown in Table 2. Also among owners the fraction of speculators is higher when the economy does well.

4.2 Simulations

To gain a better insight into the dynamics in our economy, we simulate a trading history for 1,000 artworks over 600 periods. We use the parameter values and the numerical solution described above, which leads to 62,583 transactions after the initial period. Let us first look at holding periods and gross returns aggregated across all periods. Panel (a) of Figure 3 shows the resulting distribution of holding periods (up to 15 periods). Because of the possibility of



(a) Distribution of holding periods



(b) Average gross financial returns

Figure 3: Panel (a) presents the frequency of forced and voluntary sales as a function of the holding period. Panel (b) presents the average log gross return as a function of the holding period.

a liquidity shock in each year, long holding periods are less likely than short holding periods. Moreover, the fraction of speculative transactions is much lower for longer holding periods. Panel (b) of Figure 3 shows the average log gross financial return as a function of the holding period. There is a clear negative correlation between the rate of price appreciation and the holding period, most notably over the shorter holding periods.

Let us now look at the effects of business cycles. What happens to valuations and observed prices in periods where our economy transitions from a recession to an expansion and vice versa? Table 3 shows a number of statistics averaged over all first, second, or third periods of expansions and recessions. As a benchmark, the first line shows how ρ —and thus each individual’s emotional dividend—changes when the economy enters a new state. In the second line, we show the average “capital gain” across all owners, computed as the difference between the owner’s value in the previous period and the (potentially new) owner’s value in the current period. As artworks are claims on infinite streams of future dividends, it is not a surprise that valuations do not change as much as the size of the dividends themselves.¹² This lower volatility of “capital” implies that the “dividend yield” of artworks is higher in expansions than in recessions. The next line shows

¹²The large initial change in average owner’s valuation is followed by smaller changes in the same direction if the state persists. This is because in an expansion (resp. recession) the combination of forced sales and a larger (resp. smaller) set of bidders is shifting items to higher-type (resp. lower-type) owners.

Table 3: Art market cycles

	$\bar{\omega}$			$\underline{\omega}$		
	1	2	3	1	2	3
Log Δ in ρ	+69.3%	–	–	–69.3%	–	–
Log Δ in avg. owner’s value	+10.5%	+0.5%	+0.3%	–10.3%	–0.5%	–0.4%
Log Δ in avg. price	+17.5%	–1.5%	–0.9%	–15.7%	+0.3%	–0.1%
Prob. of voluntary consignment	4.19%	2.40%	1.50%	0.05%	0.20%	0.26%
Prob. of voluntary sale	2.08%	1.24%	0.79%	0.01%	0.09%	0.12%

the mean change in average transaction prices, based on the observed trades in each period. A striking result is that observed price levels react more heavily to changes in the state of the economy than owner’s values.

The last two lines of Table 3 shows the average probability of a voluntary consignment and of a successful voluntary sale. We see a relatively high voluntary sales volume at the start of an expansion, but if the expansion continues the next periods show lower numbers of speculative trades. By contrast, we observe very few voluntary sales at the start of a recession, even if the numbers creep up if the recession continues. In other words, the existence of types who sometimes behave like collectors and sometimes like speculators leads to a positive correlation between the economy and voluntary sales volume.¹³

4.3 Endogenous Trading and Price Index Estimation

How do endogenous sale decisions affect estimates of the returns to art investors based on transaction prices? As death, debt, and divorce make all artworks in our economy trade on a regular basis, even if only at low frequency, long-term art return estimates based on observed transaction prices will not suffer from a bias. Yet, two other problems emerge when constructing a price index based on observed transactions.

First, because of the higher average appreciation rates for short holding periods, repeat-sales

¹³Yet, in our simulations, changes in total volume are mainly driven by changes in the number of forced sales, which we assume are higher in recessions.

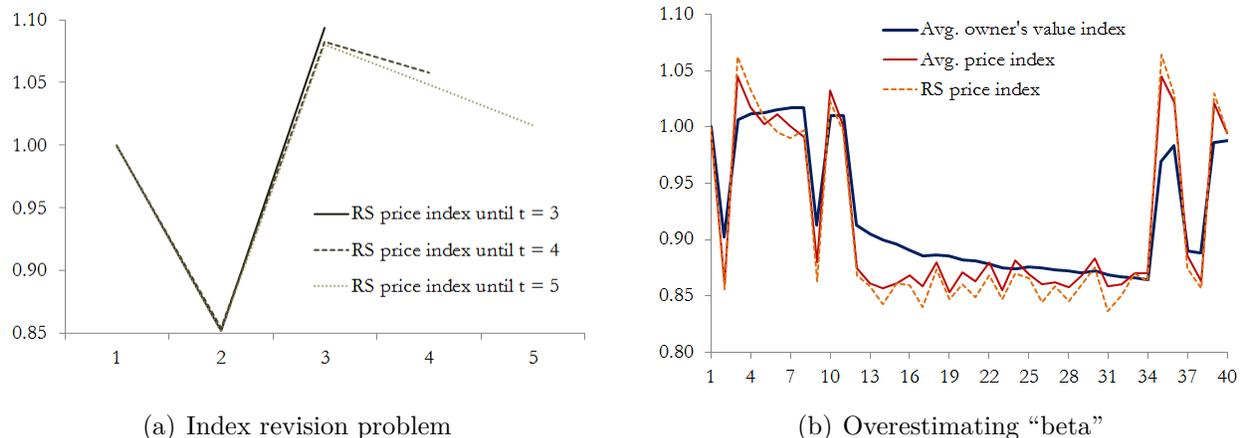


Figure 4: Panel (a) illustrates the index revision problem over the first five time periods of our simulation. Panel (b) presents an index of average owner’s value, an index of average transaction price, and an arithmetic repeat-sales price index based on all sales pairs over the first 40 time periods.

art price indexes may overestimate returns for recent periods if estimated over short horizons, or if many of the assets have not been observed (re)trading before.¹⁴ The index will need to be adjusted downwards as more data become available—an “index revision problem”. Panel (a) of Figure 4 illustrates this problem by estimating an arithmetic repeat-sales index (Shiller (1991)) over the first three, four, and five periods of our simulation. The index values in expansionary periods 3 and 4 are visibly revised downwards as more data become available. Several studies (e.g., Clapp and Giaccotto (1999), Korteweg and Sorensen (2013)) present evidence of the existence of an index revision problem in the housing market.

A second issue is the overestimation of the sensitivity—the “beta”—of art values to economic cycles by all standard transaction-based price indexes. The results in Table 3 already suggested that price indexes will overestimate the changes in valuations when the state of the economy changes. Panel (b) of Figure 4 further illustrates this problem over the first 40 periods in our simulated art market. It shows indexes for the average owner’s value across all artworks and for the average transaction price across the observed trades in each period. It also shows an

¹⁴The problem disappears if the same set of assets is trading regularly over longer periods, because then the addition of new sales pairs to the database does not increase the availability of “old” price information. For example, if an item trades in periods 1, 5, and 10, both the time 1 and time 5 prices start contributing to the estimation of the price index in period 5, while in period 10 only the contemporaneous price needs to be added.

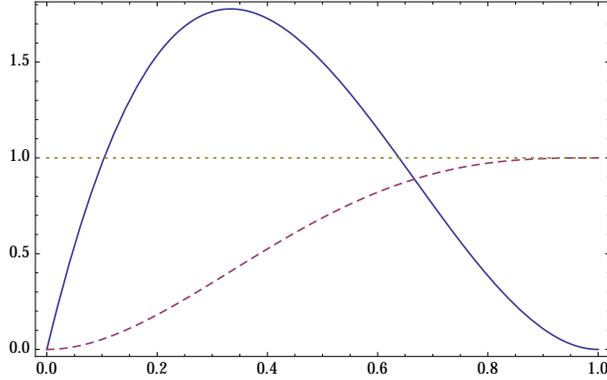


Figure 5: This figure shows the c.d.f. (dashed line) and density function (solid line) of types e under the assumption that e follows a beta distribution with parameters $\alpha = 2$ and $\beta = 3$.

arithmetic repeat-sales price index computed using all price pairs over this time frame. The indexes show unbiased long-term returns, but overestimate art’s covariance with the economy.

5 Masterpieces

We now illustrate that the distribution of tastes in the population may affect the equilibrium outcomes in the market. We introduce a category of artworks, namely “masterpieces”, for which the distribution of bidders’ tastes first-order stochastically dominates that of the artworks considered before. In other words, masterpieces are artworks for which a larger fraction of bidders can be expected to derive a higher personal enjoyment. More specifically, we numerically compute the equilibrium using the new density function for e that is shown in Figure 5.

The difference with ordinary artworks in the distribution of tastes implies that, on average, bidders’ valuations and owners’ reserve prices are higher for masterpieces. Furthermore, Table 4 shows that both in expansions and in recessions the threshold level for the emotional dividend e^* that separates collectors from speculators is slightly higher for masterpieces than for ordinary artworks. This means that, for any given level of individual-specific emotional dividend, the incentive to sell is stronger for masterpieces than for other artworks, because of the larger expected resale revenue. Yet, despite the larger e^* , the fraction of speculators among bidders in

Table 4: Recessions and expansions for masterpieces

	$\underline{\omega}$	$\bar{\omega}$
e_{ω}^*	0.19	0.35
Fraction of speculators among bidders	16.2%	46.2%
Fraction of speculators among owners	0.0%	0.0%

the masterpiece market is much lower than before, and there are virtually zero speculator-type owners. Because collectors are much more important in the market for masterpieces, the average holding period across all owners will be longer in this market.

6 Empirical Implications

6.1 Entry Prices, Holding Periods, and Financial Returns

An empirical implication of our model is that we should observe a positive correlation between purchase price and holding period, *ceteris paribus*. Because of their high emotional dividends, collectors pay relatively high prices, and only sell after a liquidity shock. By contrast, speculators pay relatively low prices, and try to resell quickly to a collector. Anecdotal evidence suggests that such a correlation indeed exists. For example, public museums, which typically have very long—even infinite—holding periods, tend to pay above-average prices in auction markets (Pommerehne and Feld (1997)). The strategy of many art investment funds, on the other hand, is to identify bargains, and to resell within a few months or years (Horowitz (2011)).

A closely related prediction, illustrated by our numerical simulations, is that there should be a negative correlation between the length of the holding period and gross financial returns for observed resales. Not only do collectors buy at higher prices and have longer holding periods on average, their consignments to auctions are also caused by liquidity shocks and therefore associated with lower reserve prices. A lower reserve price implies a lower likelihood of the item

Table 5: Holding periods and financial returns are negatively correlated

Dependent variable Model	Ln(annualized real return on resale)	
	(1)	(2)
Ln(holding period)	-0.0175*** (0.0031)	-0.0288*** (0.0035)
Purchase year fixed effects?	Yes	Yes
Artist fixed effects?	No	Yes
N	8,827	8,827
R^2	0.12	0.37

being bought in, and a lower expected price conditional on being sold.

We empirically test whether holding periods and gross returns on resales are negatively correlated, using a data set of art transaction prices. Renneboog and Spaenjers (2013) identify purchases and sales of the same work by matching transactions within the online database Art Sales Index on a number of dimensions, such as the artist, the title and dimensions of the work, etc. We take a subsample of 8,827 price pairs from their data set, focusing on transactions between 1982 and 2007.¹⁵ For each pair, we compute the annualized gross return in real USD terms.

In the first column of Table 5, we regress the logged annualized real return on the log of the holding period in years. We also include purchase year dummies, to mitigate concerns that our results are driven by the cyclical nature of the art market.¹⁶ In the second column, we repeat this regression adding artist fixed effects, so that the regression is exploiting variation in returns and holding periods between the different sales of an artist’s works. In both models, we find a significant negative relation between annualized returns and holding periods.¹⁷

¹⁵To avoid false classifications, we exclude artist-title combinations for which we observe two transactions within a 12-month period or repeated sales of works with different sizes, as these are indications that the artist made multiple works with the same title (e.g., “Still life with flowers”, “Mao”). We also exclude works on paper.

¹⁶One potential worry is that nominal loss aversion could lead buyers who buy at a market peak to hold longer (Genesove and Mayer (2001)). Another worry could be that the items purchased and sold during the last years of our time frame combine short holding periods with excellent investment performance.

¹⁷We find similar results when using the database constructed by Goetzmann, Renneboog, and Spaenjers (2011). Also Korteweg, Kräussl, and Verwijmeren (2013) report an unconditional negative correlation.

6.2 Economic Fundamentals and Cycles in the Art Market

We presented comparative statics that explained how permanent shifts in the parameters of the model impact art valuations. One of the empirical predictions is that a larger population of bidders should be associated with higher resale values and thus bids. To some extent, the impact of the inflow of collectors from emerging economies on the price of their nation's art (e.g., Renneboog and Spaenjers (2011)) can be understood from this perspective.

In our non-stationary set-up, we explored the relation between the business cycle and the art market. In line with the model's prediction, art prices have historically often gone down in recessionary periods, such as 1974–1975, the early 1980s, the early 1990s, 2001, and 2008–2009. Moreover, a number of papers have documented the positive correlation between equity returns and art prices (e.g., Hiraki et al. (2009), Goetzmann, Renneboog, and Spaenjers (2011)). The model suggests that this relation may not only be due to the impact of changes in wealth on the willingness to pay for luxury consumption (Aït-Sahalia, Parker, and Yogo (2004)), but also to the lower number of bidders and the higher likelihood of liquidity shocks in such periods.

It also follows from the model that economic expansions (or positive shocks to the number of bidders) lead to an increase in the threshold emotional dividend level that separates collectors from speculators. As a consequence, some long-term collectors may voluntarily decide to resell—*i.e.*, to become speculators. The model thus implies a positive correlation between relative prices and the volume of voluntary consignments. It is well-known that a relation between price and total volume indeed exists; Figure 6 shows the log real USD returns and the number of observed transactions per year, over the 1982–2006 period, from Renneboog and Spaenjers (2013).

The relative importance of speculative sales in total volume is unobservable. However, if forced sales are associated with lower reserve prices, buy-in rates should move in line with the fraction of voluntary consignments. Ashenfelter and Graddy (2011) show that buy-in rates spike up at the end of booms, which suggests that speculative trading is widespread in such periods.

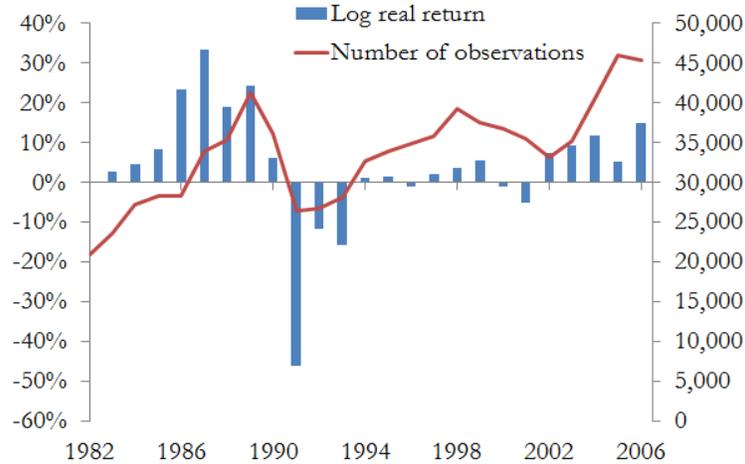


Figure 6: This figure shows the log real USD returns (against the left axis) and total number of transactions (against the right axis) per year from Renneboog and Spaenjers (2013).

6.3 Masterpieces

A final empirical prediction of the model that we consider here is that holding periods should be longer on average for masterpieces than for other artworks. This prediction seems to be supported by the often-heard complaint that the market for high-quality works gets “thinner” over time, as more and more masterpieces end up in permanent (institutional) collections. To see whether we can also find evidence for differences in average holding periods among the items that do (re)trade, we use the newly constructed database of transaction pairs over the period 1982–2007 presented before. In Table 6, we regress log holding periods against a number of proxies for an artwork’s quality, taken from Renneboog and Spaenjers (2013).

In the first column, we use a dummy variable that equals one if the artist was included in the art history textbook “Gardner’s Art Through the Ages” at the time of the purchase. In the second column, we include a continuous variable measuring the length (*i.e.*, the log of the number of words) of the artist’s biography in the online art history database Grove Art Online. In the third column, we use a dummy indicating whether the artwork was purchased at the London or New York offices of Sotheby’s or Christie’s. In all models, we include purchase year fixed effects, as holding periods are by construction shorter for later purchase year. We thus

Table 6: Holding periods are longer for masterpieces

Dependent variable	Ln(holding period)			
	(1)	(2)	(3)	(4)
Model				
Sample period	1982–2007	1982–2007	1982–2007	1982–2007
Artist in textbook	0.0943*** (0.0254)			
Length of artist’s biography		0.0112* (0.0059)		
Purchase at large auction house			0.1728*** (0.0171)	0.1633*** (0.0245)
Purchase year fixed effects?	Yes	Yes	Yes	Yes
Artist fixed effects?	No	No	No	Yes
N	8,827	8,827	8,827	8,827
R^2	0.14	0.14	0.15	0.37

exploit variation in holding periods between items that have been purchased in the same year (and that have been sold before the end of 2007). In the fourth column, we repeat the third model, but now adding artist fixed effects. We see a very robust positive relation between our proxies for quality and the average holding period, in line with our model’s prediction.

7 Possible Extensions of the Model

7.1 Private Transactions and the Existence of Investment Funds

In our economy there is no role for a professional speculator who does not enjoy any emotional dividend. Indeed, the expected equilibrium utility of an agent with $e = 0$ equals zero despite the high expected financial returns conditional on acquisition, because this individual never wins an auction. In reality, however, a private art market exists alongside the auction market. Moreover, selling privately is often considered as the only option for owners who need cash quickly. Some art funds claim to exploit inside market knowledge on such distressed owners.

The introduction of a private market with an opportunistic investor would not affect the equilibrium in our model. Consider a well-connected art investment fund that is able to identify

and approach a distressed art owner. If this fund privately offers $R(0) + \epsilon$ in exchange for the artwork, the owner will accept the take-it-or-leave-it offer, because the expected revenues from auctioning equal $R(0)$. The private buyer can then flip the artwork at auction by choosing the optimal reservation price associated with a type $e = 0$, namely $r(0)$. For ϵ arbitrarily small, the presence of such art funds in the market would not change expected cash flows from resale, and as a result the equilibrium behavior of bidders and owners would not be affected.

7.2 Endogenous Entry and Exit of Bidders

In our benchmark model we simply assumed that there was a renewal of n bidders at every auction. We now sketch how entry and exit could be endogenized by considering agents' utility from not buying the artwork. Suppose that an agent, instead of buying the artwork, can immediately acquire a non-durable consumption good that provides an enjoyment of c . When an artwork comes up for auction, an individual will then only bid if the expected payoff conditional on winning the artwork, *i.e.*, $\frac{U(e)}{F^n(e)}$, exceeds c . Now consider a bidder who has just lost the auction. If he lost against a collector, then he knows that the winner can be expected to resell in $1/d$ periods only. If he lost against a speculator, then the expected horizon until his next chance to buy will be even longer, as the "loser" is himself a speculator and the winning speculator will only voluntarily resell to collectors. Furthermore, the losing bidder will only have a new opportunity to buy the artwork if he is not hit by a liquidity shock before the artwork is back on the market. A loser's expected payoff from saving his resources until the next auction is therefore at most $\frac{\delta d(1-d)U(1)}{1-\delta(1-d)^2} < U(1)$. If c exceeds this threshold, the loser will prefer spending his resources on the alternative form of consumption and exiting the artwork market. To summarize, if $\frac{\delta d(1-d)U(e)}{1-\delta(1-d)^2} < c < \frac{U(e)}{F^n(e)}$, an agent of type e will participate in the auction but disappears from the market if not being the winning bidder.

One could endogenize the number of bidders in a more general fashion by starting from a

larger population of N agents who are aware that the artwork exists. An agent's type is the couple (e, c) . A fraction d of N is renewed exogenously in every period. When the artwork is auctioned, only those agents for whom $U(e)$ is relatively large compared to c will enter the auction. This condition may not be satisfied for a large fraction of the population if many individuals find it optimal to spend their time and money otherwise (e.g., on basic consumption). Moreover, if agents cannot finance art purchases through debt, then only individuals with investible wealth of at least $b(0) > 0$ will be able to participate in the auction. In other words, the resale possibility can make the optimal bid of a low-type individual substantially exceed his financial capacity. Endogenizing the number of bidders in this way may correspond to having a mass of agents with $e = 0$ in our model.

7.3 Substitutes

What if items are not truly unique, but a number of substitutes exist? This would be equivalent to introducing an “outside option” with a value that is increasing in e . As a result, bids will be lowered, and more so for the high types. Because of the decrease in the sensitivity of the bidding function to e , there will be less scope for speculation. If many substitutes exist (e.g., prints, stamps), the willingness to pay for any individual item will only weakly increase with one's type, and there will be little speculative behavior in the auction market.

7.4 Temporal Variation in Tastes

We assumed that owners' emotional dividends can change with the macroeconomic state, through the factor ρ_{ω_t} . Whereas variation in this parameter can also be interpreted as shifts in tastes in the population of future bidders, we assumed the taste of any owner e_i to be fixed. The model can be extended to allow e_i to vary over time. An owner's decision whether to sell would then depend not only on the current emotional dividend level but also on the conditional

expectation of future emotional dividends. How such variability in enjoyment would affect bids, sale decisions, and reserve prices depends on the the assumed evolution of e_i . Assuming that e_i tends to decrease (e.g., because the novelty of owning an artwork wears off) would decrease bids and increase the volume of voluntary sales. The opposite would happen if e_i tends to increase over time (e.g., because owners become more attached to the artwork over time).

7.5 Uncertainty about Tastes

We assumed that the distribution of tastes follows a common-knowledge F . Although this may be a reasonable assumption for artworks with a trading history spanning multiple centuries, it is less realistic for contemporary art. What if we introduce uncertainty about the distribution of types? Agents will then update their beliefs about the distribution of tastes based on observed auction outcomes. A high price (resp. buy-in) makes agents believe that the true F is closer to (resp. further away from) that of “masterpieces” than that of “ordinary” art. The updated beliefs about resale opportunities lead to an upward shift in $b(e)$, which in itself raises price levels. However, also reserve prices move upwards, and therefore buy-ins become more likely, which may reverse the process. We believe that such an extension raises interesting questions and technical challenges that motivate further study.

8 Conclusion

We presented a stylized model of trading in an infinite-horizon economy in which individuals exchange and enjoy art. The value of an artwork to its owner depends on both an idiosyncratic personal pleasure component and a financial component. The latter value is endogenously related to the distribution of tastes in the population. In a stationary economy, two types of art buyers emerge in equilibrium: “collectors”, who have a large private use value of the artwork and sell only if hit by a liquidity shock, and “speculators”, who do not derive much enjoyment

from ownership and try to resell quickly. Speculators can expect larger financial returns than collectors, but it is the collectors with the highest emotional dividends that achieve the highest levels of equilibrium utility and are most likely to own an artwork. Once we assume that business cycles affect our market, there will also be owners who would hold on the artwork during recessions, but would sell in an expansion. The model produces a rich set of empirical predictions that find support in art transaction data. The model also highlights how the endogeneity of sale decisions may lead to sample selection problems with transaction-based price indexes in these markets. More specifically, repeat-sales price indexes may suffer from “index revision problems”, while all price indexes will overestimate the sensitivity of the value of the underlying portfolio of assets to economic cycles.

Although our model is designed to reflect the institutional framework of the auction market for art and other collectibles, its logic can be applied to a number of other markets. The crucial features of the economy that we consider can be summarized as follows. First, economic agents trade in unique durable assets. Uniqueness also implies that short sales are impossible. Second, agents derive idiosyncratic and non-tradable dividends from the assets. Third, there is a pool of potential buyers at any date, but because of transaction costs—even for unsuccessful trades—an owner will only infrequently offer his item for sale through an auction-like mechanism. One example of a market with these features is the luxury real estate market, as there is variation in individuals’ preferences for the ownership of certain housing characteristics. Another example is the intercorporate asset sales market. Following a liquidity shock or change in strategy, a company may decide to sell a subsidiary. Each potential buyer’s valuation is a function of the idiosyncratic industrial synergies with the divested assets. Managers of bidding companies must however also take into account that in the future they could themselves be willing—or be forced—to sell.

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Appendix

Proof of Proposition 1

Note that in an STM equilibrium, an individual's strategy only depends on the individual's type and on the state of the economy. Hence we can replace the time subscripts in equations (1), (3), and (4) with the state ω at time t to obtain the following equations:

$$\begin{aligned}
 R_\omega(r) &= (1 - \gamma_1)E[\max\{b_\omega(\tilde{e}^{(2)}), r\} | b_\omega(\tilde{e}^{(1)}) \geq r, \omega] \Pr[b_\omega(\tilde{e}^{(1)}) \geq r | \omega] - \gamma_0 r \Pr[b_\omega(\tilde{e}^{(1)}) < r | \omega] \\
 &= n_\omega(1 - \gamma_1) \left(\int_{e_\omega(r)}^1 (n_\omega - 1)b_\omega(z)f(z)F(z)^{n_\omega-2}(1 - F(z))dz + rF(e_\omega(r))^{n_\omega-1}(1 - F(e_\omega(r))) \right) \\
 &\quad - \gamma_0 F(e_\omega(r))^{n_\omega}
 \end{aligned} \tag{6}$$

$$V_\omega(e) = \max\{b_\omega(e), \max_r R_\omega(r) + b_\omega(e)F(e_\omega(r))^{n_\omega}\} \tag{7}$$

$$b_\omega(e) = \rho_\omega e + \delta \sum_{\omega' \in \Omega} p_\omega(\omega') (d_{\omega'} R_{\omega'}(0) + (1 - d_{\omega'}) V_{\omega'}(e)), \tag{8}$$

where $e_\omega(r) := b_\omega^{-1}(r)$, *i.e.*, the bidder type who would bid exactly r in state ω .

Let B be the space of bounded, continuous, and non-decreasing functions $g : [0, 1] \rightarrow \mathbb{R}$, with the sup norm $\eta : B \rightarrow \mathbb{R}$, *i.e.*, $\eta(g) := \sup_{e \in [0, 1]} |g(e)|$. A state-contingent bidding strategy is a point b in the product space $B^\Omega := \prod_{\omega \in \Omega} B$ with the sup norm $\|b\| := \max_\omega \eta(g_\omega)$. Fix some time t and a state-contingent bidding strategy $b \in B^\Omega$, and consider a time t bidder of type e who believes that in any future period $\tau > t$: (i) bidders state-contingent bids will be b , (ii) his continuation payoff from not selling in period τ is $b_{\omega_\tau}(e)$. Given these beliefs, denote by $R_{\omega_\tau}(r)[b]$ and $V_{\omega_\tau}(e)[b]$ the expected resale revenue from auctioning at reserve price r and the expected continuation payoff from owning the artwork at some future time $\tau > t$, respectively. The quantities $R_\omega(r)[b]$ and $V_\omega(e)[b]$ are equal to the r.h.s. of (6) and (7), respectively. Then,

the maximum bid of such a bidder in t is:

$$(\mathcal{T}_{\omega_t} b)(e) := \rho_{\omega_t} e + \delta \sum_{\omega' \in \Omega} p_{\omega_t}(\omega') (d_{\omega'} R_{\omega'}(0)[b] + (1 - d_{\omega'}) V_{\omega'}(e)[b])$$

Let \mathcal{T} be the operator associating to any state-contingent bidding function $b \in B^\Omega$ the state-contingent function $\{\mathcal{T}_\omega b\}_{\omega \in \Omega}$. Clearly a STM equilibrium equilibrium is a fixed point of the operator \mathcal{T} . In what follows, we will use the contraction mapping fixed point theorem to prove existence and uniqueness of a STM equilibrium. Namely we will prove that there exists $\beta \in (0, 1)$ such that for any $b, b' \in B^\Omega$, $\|\mathcal{T}b - \mathcal{T}b'\| \leq \beta \|b - b'\|$.

Let us introduce some notation. For any $g, g' \in B$, if $g(e) \leq g'(e)$ for all $e \in [0, 1]$, we write $g \leq g'$. For any function $g \in B$ and $n \in \mathbb{N}$, let define the functions $R_{g,n}(e_r)$ as the expected cashflow for an owner who auctions the artwork for a reserve price equal to $g(e_r)$ and who believes that there are n bidders and that their bidding function at the auction is g . This is the same as $R_\omega(g(e_r))[b]$ when $b_\omega = g$ and $n_\omega = n$. Formally,

$$\begin{aligned} R_{g,n}(e_r) &:= (1 - \gamma_1) E[\max\{g(\tilde{e}^{(2)}), g(e_r)\} | \tilde{e}^{(1)} \geq e_r] \Pr(\tilde{e}^{(1)} \geq e_r) - \Pr(\tilde{e}^{(1)} < e_r) \gamma_0 g(e_r) \quad (9) \\ &= n(1 - \gamma_1) \left(\int_{e_r}^1 (n-1) g(z) f(z) F(z)^{n-2} (1 - F(z)) dz + g(e_r) F(e_r)^{n-1} (1 - F(e_r)) \right) \\ &\quad - \gamma_0 g(e_r) F^n(e_r) \end{aligned}$$

Let $Q_{g,n}(e, e_r)$ denote such an owner's expected continuation payoff if he values the asset at $g(e)$ and decides to auction it with reserve price $g(e_r)$. Formally,

$$Q_{g,n}(e, e_r) := R_{g,n}(e_r) + F(e_r)^n g(e) \quad (10)$$

Let $\mathcal{K} : B \rightarrow B$ be the operator that associates to a function $g \in B$ the function $\mathcal{K}g : [0, 1] \rightarrow$

\mathbb{R} as follows:

$$\mathcal{K}g(e) := \max\{g(e), \max_{e_r \in [0,1]} Q_{g,n}(e, e_r)\} \quad (11)$$

Note that the function $\mathcal{K}g(e)$ is the same as $V_\omega(e)[b]$ when $b_\omega = g$ and $n_\omega = n$. The proof includes two lemmas.

Lemma 1 *For any couple g, g' in B , it results that $\eta(\mathcal{K}g - \mathcal{K}g') \leq \eta(g - g')$ and $\mathcal{K}g \in B$.*

Proof: First we show that \mathcal{K} satisfies the following two conditions:

- a. Monotonicity: $g \leq g' \in B$ implies $(\mathcal{K}g)(e) \leq (\mathcal{K}g')(e)$, for all $e \in [0, 1]$.
- b. Scaling: for any $g \in B$ and constant $a \geq 0$:

$$\eta(\mathcal{K}(g + a) - \mathcal{K}g) \leq a,$$

where $(g + a)(e) := g(e) + a$.

Let us consider monotonicity. Take any $g, g' \in B$ such that $g \leq g'$ and suppose that there exists e such that $(\mathcal{K}g)(e) > (\mathcal{K}g')(e)$. Because $g(e) \leq g'(e)$, it cannot be that $(\mathcal{K}g)(e) = g(e)$ and $(\mathcal{K}g')(e) = g'(e)$. Nor can it be that $(\mathcal{K}g')(e) \neq g'(e)$ and $(\mathcal{K}g)(e) = g(e)$, because in this case $g'(e) \geq g(e) = (\mathcal{K}g)(e) > (\mathcal{K}g')(e)$ would contradict the relation $(\mathcal{K}g')(e) \geq g'(e)$ that is implied by the definition of \mathcal{K} (equation (11)). Hence it must be that $(\mathcal{K}g)(e) \neq g(e)$, implying $(\mathcal{K}g)(e) = \max_{e_r \in [0,1]} Q_{g,n}(e, e_r)$. Let e_r^* be such that $(\mathcal{K}g)(e) = Q_{g,n}(e, e_r^*)$ and let $r^* := g(e_r^*)$.

Observe that:

$$g(e) < Q_{g,n}(e, e_r^*) = (1 - \gamma_1)E[\max\{g(\tilde{e}^{(2)}), r^*\} | g(\tilde{e}^{(1)}) \geq r^*] \Pr(g(\tilde{e}^{(1)}) \geq r^*) + \Pr(g(\tilde{e}^{(1)}) < r^*)(g(e) - \gamma_0 r^*),$$

where the equality comes from (6), (10), and (11), whereas the inequality comes from the fact that $(\mathcal{K}g)(e) \neq g(e)$ and implies:

$$(1 - \gamma_1)E[\max\{g(\tilde{e}^{(2)}), r^*\} | g(\tilde{e}^{(1)}) \geq r^*] > (g(e) - \gamma_0 r^*) \quad (12)$$

Let e'_r be such that $g'(e'_r) = r^*$. Because $g' \geq g$, we have:

$$\Pr(g(\tilde{e}^{(1)}) \geq r^*) \leq \Pr(g'(\tilde{e}^{(1)}) \geq r^*) \quad (13)$$

$$\Pr(g(\tilde{e}^{(1)}) < r^*) \geq \Pr(g'(\tilde{e}^{(1)}) < r^*) \quad (14)$$

Inequalities (12), (13), and (14) imply the first inequality below, whereas the second and third inequalities come from $g \leq g'$:

$$\begin{aligned} (\mathcal{K}b)(e) &= \\ Q_{g,n}(e, e_r^*) &\leq (1 - \gamma_1)E[\max\{g(\tilde{e}^{(2)}), r^*\} | g(\tilde{e}^{(1)}) \geq r^*] \Pr(g'(\tilde{e}^{(1)}) \geq r^*) + \Pr(g'(\tilde{e}^{(1)}) < r^*)(g(e) - \gamma_0 r^*) \\ &\leq (1 - \gamma_1)E[\max\{g(\tilde{e}^{(2)}), r^*\} | g(\tilde{e}^{(1)}) \geq r^*] \Pr(g'(\tilde{e}^{(1)}) \geq r^*) + \Pr(g'(\tilde{e}^{(1)}) < r^*)(g'(e) - \gamma_0 r^*) \\ &\leq (1 - \gamma_1)E[\max\{g'(\tilde{e}^{(2)}), r^*\} | \tilde{g}'(e^{(1)}) \geq r^*] \Pr(g'(\tilde{e}^{(1)}) \geq r^*) + \Pr(g'(\tilde{e}^{(1)}) < r^*)(g'(e) - \gamma_0 r^*) \\ &= Q_{g'}(e, e'_r) \leq (\mathcal{K}g')(e). \end{aligned}$$

Hence a contradiction that $(\mathcal{K}g)(e) > (\mathcal{K}g')(e)$.

Now let us consider scaling. Let $g, g' \in B$ such that for any $e \in [0, 1]$, $g'(e) = g(e) + a$, where a is a positive constant. For any quadruple $e^1 \geq e^2$, e and e_r in $[0, 1]$, let $Q_{g,n}(e, e_r)(e^1, e^2)$ be the value of $Q_{g,n}(e, e_r)$ when the highest bidder type is e^1 and the second highest bidder type is e^2 . Namely:

$$Q_{g,n}(e, e_r)(e^1, e^2) = \begin{cases} -\gamma_0 g(e_r) + g(e) & \text{if } e^1 < e_r \\ (1 - \gamma_1)g(e_r) & \text{if } e^2 \leq e_r < e^1 \\ (1 - \gamma_1)g(e^2) & \text{if } e_r < e^2 \end{cases}$$

Note that $g'(e) = g(e) + a$ implies:

$$Q_{g',n}(e, e_r)(e^1, e^2) = \begin{cases} Q_{g,n}(e, e_r)(e^1, e^2) + (1 - \gamma_0)a & \text{if } e^1 < e_r \\ Q_{g,n}(e, e_r)(e^1, e^2) + (1 - \gamma_1)a & \text{if } e_r \leq e^1 \end{cases} \quad (15)$$

Because $Q_{g,n}(e, e_r) = E[Q_{g,n}(e, e_r)(\tilde{e}^{(1)}, \tilde{e}^{(2)})]$, it follows from equation (15) that for any e and e_r , $Q_{g',n}(e, e_r) = Q_{g,n}(e, e_r) + a(1 - \gamma_0 \Pr(\tilde{e}^{(1)} < e_r) - \gamma_1 \Pr(\tilde{e}^{(1)} \geq e_r)) \leq Q_{g,n}(e, e_r) + a$ and hence $\eta(Q_{g,n}(e, e_r) - Q_{g',n}(e, e_r)) \leq a$, for any e and e_r . As a consequence:

$$\eta(\max_{e_r \in [0,1]} Q_{g,n}(e, e_r) - \max_{e_r \in [0,1]} Q_{g',n}(e, e_r)) \leq a \quad (16)$$

Considering that $\eta(g - g') = a$ and the definition $\mathcal{K}g$ in (11), we can conclude that $\eta(\mathcal{K}g' - \mathcal{K}g) \leq a$.

We can prove now the first statement of the lemma. Note that for any $g, g' \in B$, we have $g \leq g' + \eta(g - g')$. Monotonicity and scaling of \mathcal{K} imply:

$$\mathcal{K}g \leq \mathcal{K}(g' + \eta(g - g')) \leq \mathcal{K}g' + \eta(g - g')$$

Because $g' \leq g + \eta(g - g')$, we also have:

$$\mathcal{K}g' \leq \mathcal{K}g + \eta(g - g')$$

Combining these two inequalities, we find that for any $e \in [0, 1]$, $|\mathcal{K}g(e) - \mathcal{K}g'(e)| \leq \eta(g - g')$, or $\eta(g - g') \geq \sup_e |\mathcal{K}g(e) - \mathcal{K}g'(e)| = \eta(\mathcal{K}g - \mathcal{K}g')$, as was to be shown. To prove the second statement we need to show that if $g \in B$, then $(\mathcal{K}g)(e)B$ is finite valued, continuous, and non-decreasing in e . Note first that $Q_{g,n}(e, e_r)$ is a finite valued for any $e, e_r \in [0, 1]$. It is also continuous in e_r and e and increasing in e , and therefore $\max_{e_r} Q_{g,n}(e, e_r)$ is also continuous and increasing in e . Hence $\max_{e_r} Q_{g,n}(e, e_r) \in B$. Because $\mathcal{K}g$ is the maximum of two functions in B , it must be in B . ■

Lemma 2 *For any couple b, b' in B^Ω , it results that $\eta(\mathcal{T}_\omega b - \mathcal{T}_\omega b') \leq \delta \|(b - b')\|$.*

Proof: Note first that for any $\omega \in \Omega$, it results that $\eta(R_\omega(0)[b] - R_\omega(0)[b']) \leq \eta(b_\omega - b'_\omega)$. Also

observe that $V_\omega(v)[b] = \mathcal{K}b_\omega$. Hence Lemma 1 implies that $\eta(V_\omega(v)[b] - V_\omega(v)[b']) \leq \eta(b_\omega - b'_\omega)$.

Thus:

$$\begin{aligned}
\eta(\mathcal{T}_\omega b - \mathcal{T}_\omega b') &= \sup_{e \in [0,1]} |\mathcal{T}_\omega b(e) - \mathcal{T}_\omega b'(e)| \\
&\leq \delta \sup_{e \in [0,1]} \sum_{\omega' \in \Omega} p_\omega(\omega') (d_{\omega'} |R_{\omega'}(0)[b] - R_{\omega'}(0)[b']| + (1 - d_{\omega'}) |V_\omega(v)(e)[b] - V_\omega(v)(e)[b']|) \\
&\leq \delta \sum_{\omega' \in \Omega} p_\omega(\omega') (d_{\omega'} \eta(b_{\omega'} - b'_{\omega'}) + (1 - d_{\omega'}) \eta(b_{\omega'} - b'_{\omega'})) = \delta \sum_{\omega' \in \Omega} p_\omega(\omega') \eta(b_{\omega'} - b'_{\omega'}) \\
&\leq \delta \sum_{\omega' \in \Omega} p_\omega(\omega') \max_{\omega''} \eta(b_{\omega''} - b'_{\omega''}) = \delta \|b - b'\|,
\end{aligned}$$

because $\sum_{\omega' \in \Omega} p_\omega(\omega') = 1$. ■

Because $\|\mathcal{T}b - \mathcal{T}b'\| = \max_{\omega \in \Omega} \eta(\mathcal{T}_\omega b - \mathcal{T}_\omega b')$, Lemma 2 implies that $\|\mathcal{T}b - \mathcal{T}b'\| < \delta \|b - b'\|$.

Thus \mathcal{T} is a contraction mapping because $\delta < 1$. Note that $\mathcal{T}_\omega b \in B$, because it is a positive linear combination of functions in B . Hence, $\mathcal{T}b \in B^\Omega$. Thus \mathcal{T} is a contraction mapping from B^Ω into B^Ω and hence has a unique fixed point. ■

Proof of Proposition 2

First note that, because V does not depend on t , any owner's equilibrium behavior is either that of a collector or that of a speculator.

1. Consider an owner of type $e = 0$. Such an individual, who does not enjoy any emotional dividend, will find it optimal to try to sell immediately. However, if the highest bid in the first auction is too low, he will prefer “buying in” and delaying the sale by one period, as the expected payoff from a sale in the next period is at least $\delta R(0)$. By continuity, also owners who enjoy a positive but relatively small emotional dividend will consign it to auction as soon as possible.

Consider now an owner of type $e = 1$. Equation (4) implies that such an owner will prefer keeping the artwork over selling for a revenue below $b(1)$. Because the maximum possible bid is exactly $b(1)$, the maximum revenue from an auction cannot exceed $(1 - \gamma_1)b(1)$, implying that such an owner will sell only if hit by a liquidity shock. The same reasoning applies for any owner with sufficiently large e .

The threshold level e^* corresponds to an owner who is indifferent between selling immediately (with an optimal reserve price) and selling once hit by a liquidity shock (without a reserve).

2. If the seller is of type z , then a winner of type e is expected to pay:

$$P_0(e, z) := dE[b(\tilde{e}^{(2)})|\tilde{e}^{(2)} \leq e] + (1 - d)E[\max\{b(\tilde{e}^{(2)}), b(\underline{e}(z))\}|\tilde{e}^{(2)} \leq e]1_{\{\underline{e}(z) < e, z < e^*\}}$$

where the first term is the payment when the sale results from a liquidity shock, and the second term is the payment when the sale is voluntary. A voluntary transaction occurs only if the owner's type z is below e^* (*i.e.*, if the owner is a speculator) and if the winning type is willing to pay at least the owner's reservation price $r(z) = b(\underline{e}(z))$ (*i.e.*, if $e > \underline{e}(z)$). Note that both terms are increasing in e . Thus, $P_0(e) = E[P_0(e, \tilde{z})]$ increases with e .

3. Consider a speculative owner of type e who is not hit by a liquidity shock, and let

$r(e)$ denote the reserve price chosen by this owner. Note that $r(e) \in [b(0), b(1)]$. Because $\Pr[b(0) \leq b(\tilde{e}^{(1)}) \leq b(1)] = 1$, a reserve price below $b(0)$ would provide the same payoff of $b(0)$, while a reserve price greater than $b(1)$ would provide a strictly negative payoff of $-\gamma_0 r(e)$. $r(e)$ is then an interior solution of the program $\max_r [R(r) + F(b^{-1}(r))^n b(e)]$, and so it satisfies the first order condition $R'(r) + nF(b^{-1}(r))^{n-1} b(e) / b'(b^{-1}(r))|_{r=r(e)} = 0$. Because the l.h.s. of this equality is increasing in e and the second order condition must be satisfied, $r(e)$ must be locally increasing in e .

In the case of a forced sale, the seller cannot afford to set a positive reserve price.

4. It sufficient to observe that in any period t the probability of the owner of type e sells is d for a collector and $d + (1 - d)Pr[b(\tilde{e}^{(1)}) > r(e)]$ for a speculator.

5. Observe that a bidder of type e has his unique opportunity to buy the artwork. His valuation of the artwork is $b(e)$, which is a strictly increasing function of e . If $b(e) > r$, in a symmetric equilibrium he will win only if the highest competitor type is not above e . Hence the equilibrium payoff has the same qualitative properties as the equilibrium payoff of a standard second-price auction and the result follows from the envelope theorem.

6. Let \tilde{e}_t denote the type of the owner at time t . Note that, for any set $x \in [0, 1]$, $\mathcal{P}G_t(x) = \Pr(\tilde{e}_{t+1} < x) = \Pr(\text{the artwork is sold at } t, h^t) \Pr(\tilde{e}^{(1)} < x) + [1 - \Pr(\text{the artwork is sold at } t, h^t)] \Pr(\tilde{e}_t < x)$. Because for any history h^t , $\Pr(\text{the artwork is sold at } t, h^t) \geq d$ and $\Pr(\tilde{e}^{(1)} < x) = F(x)^n$, the only fixed point of \mathcal{P} is $G^*(x) = F(x)^n$. ■