The 3 C's of Outsourcing Innovation: Control, Capability, and Cost

Firms are increasingly outsourcing design, in addition to manufacturing. In a two-stage supply chain, we investigate the impact of control (who has decision rights on the process and product innovation investment), capability (who is better at the innovation activity), and cost (who can perform the innovation activity more economically) on the outsourcing decision, the investment in innovation, and the resulting supply chain performance. The buyer determines whether to implement process innovation (in the form of cost reduction) and/or product innovation (in the form of increased value to consumers) and whether to outsource either (or both) of these activities to the supplier. We consider three cases: no outsourcing, outsourcing manufacturing and process innovation, and outsourcing both manufacturing/process innovation and design (product innovation). We show that when the supply chain uses a wholesale pricing contract, the buyer outsources manufacturing (and process innovation) when the supplier is much better in terms of capability and/or cost than the buyer; however, the buyer will never outsource both process and product innovation. On the other hand, when a two-part tariff contract is used, the buyer will outsource both manufacturing and design as long as the supplier is better in one of the parameters (cost or process/product innovation capability) and equal in the other two, and this outsourcing will result in a higher supply chain innovation investment. We also find that firms with functional products will, in general, invest more in process innovation than in product innovation, while the opposite is true for innovative products. Lastly, we examine the impact of demand uncertainty on outsourcing innovation and show that when outsourcing both process and product innovation, uncertainty requires the supplier to have a higher process innovation capability advantage (compared to the deterministic case) when her cost is higher than the buyer’s and a lower capability advantage when it is lower.

1. Introduction

Increasingly, companies are outsourcing product innovation (design) in addition to manufacturing and the associated process innovation (Deutsch 2004, Stanko and Calantone 2011). This trend is widespread across many industries, where suppliers are becoming the major contributors of innovations in the supply chain. In the electronics industry, Cisco Systems Inc. uses partners to provide hardware and manufacturing innovation (Quinn 2000). Contract manufacturers like Sanmina Corporation and Jabil Circuit Inc. have increased design departments in response to this demand from their customers (Deutsch 2004). For example, Sanmina recently designed and manufactured products for several different customers, including a mobile hotspot for Uros, a telecommunication company, and a medical device to check for ear infections for Innova Medical (Sanmina 2013).

Consider the case of Ford Motor Company. Ford began as a vertically integrated firm in the time of Henry Ford (Gelderman 1981). Today at Ford, not only is manufacturing of major subassemblies outsourced, but also the design of those subassemblies. For example, in the late 1980s, Ford outsourced the development of subcompacts to Mazda and used Mazda’s lean processes expertise to build a new production facility in Mexico (Welch and Nayak 1992). In 2009, Ford’s third quarter success was attributed in part to its outsourcing design and manufacturing of its electric vehicle powertrains to companies such as Magna International and Azure Dynamics (Motavalli 2009). By outsourcing, Ford utilizes its suppliers’ specialized capabilities in electric vehicles, and focuses its resources on other parts.
of its business. The potential benefits of outsourcing production and innovation include access to low cost labor, access to innovative ideas and technology, and reduction in development time, while the potential drawbacks include intellectual property (IP) loss, supplier opportunism, and increased supply chain risk (Carson 2007, Amaral et al. 2011). Boeing’s woes with the 787 illustrate many of the risks of outsourcing design and manufacturing, such as parts that did not fit together and poor communication that led to severe budget overruns and project delays (Denning 2013). Abstracting away from the numerous dimensions of outsourcing, we focus on the impact of firms’ innovation costs and capabilities on outsourcing and the investments in product and process innovation. We therefore address two key questions for companies such as Ford: First, when should a firm outsource manufacturing versus both manufacturing and design, and second, what is the impact of outsourcing on the investments in process and product innovation?

Process innovation is defined in this paper as an activity that results in cost reduction, while product innovation results in an increase in consumer value. We examine the process and product innovation decisions in a supply chain using a stylized model and comparing three possible scenarios: 1) the firm manufactures and performs process and product innovation in-house, 2) the firm outsources manufacturing and process innovation, and 3) the firm outsources both manufacturing (including process innovation) and product innovation. When the firm outsources one or both of these functions, it is relying on its supplier for commercial success. In particular, the supplier then makes innovation-focused investments in order to reduce production cost, or improve product value. For example, auto makers (such as Ford) that outsource subsystems design and production rely on suppliers for product improvement proposals and manufacturing cost reduction suggestions (Takeishi 2001). We assume the supplier recoups the costs of these investments through the wholesale/transfer price charged. We focus on three key drivers of outsourcing innovation: control, capability, and cost, which we call the 3C’s of outsourcing innovation.

The first “C”, Control, refers to which firm has decision rights regarding the process and product innovation investment. There is a vast supply chain literature (e.g., Cachon 2003, Liu et al. 2007, etc.) focusing on the issue of supply chain control and the question of which firm in the supply chain controls the production (inventory) decision. Within this literature, outsourcing manufacturing means that the production decisions are decentralized, resulting in double marginalization and a lower order quantity, and thus, centralized control of the ordering/inventory decision is optimal. In this paper, outsourcing manufacturing means the process innovation decision is controlled by the supplier and product innovation by the buyer, while outsourcing manufacturing and design means both the process and the product innovation decisions are controlled by the supplier.

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1 We use innovation and improvement interchangeably, where improvement would be a minor innovation, thus abstracting away from the degree of innovation.
The second “C”, Capability, refers to how effective the firm is at implementing the innovation decision, drawing on the literature about core competencies (e.g., Prahalad and Hamel 1990) and capabilities (e.g., Barney 1991, Argyres 1996). Given a firm's (and its employees’) past experiences, its accumulated stock of competencies or its better information may make it more effective than another firm at certain aspects of business, in this case innovation (Carrillo and Gaimon 2004, Huang et al. 2009). In the resource-based view theory, firms are heterogeneous with respect to these capabilities, and these capabilities tend to be hard to imitate, thus a firm may be unable to simply invest to obtain the same capability as another firm or (in the case of our paper) its supplier (Leiblein and Miller 2003). Alternatively, one could interpret capability as a firm being specialized, and thus more effective than its customer at process and/or product innovation. For example, Johnson Controls, which manufactures subsystems for several automotive customers, has created capabilities in these subsystems that its individual customers would likely not be able to develop or maintain due to its scale and specialization (Quinn 2000). In our model, we allow either firm to be more effective at either process innovation (reducing cost), product innovation (increasing value), or both.

The last “C” is Cost. One of the main reasons firms choose to outsource either manufacturing (and the resulting process innovation) or product innovation is to reduce costs (e.g., Engardio and Einhorn 2005, Gray et al. 2009b, Wu and Zhang 2013, Feng and Lu 2012a). Firms typically incur lower variable costs and reduce fixed costs by outsourcing. In this paper, firms face different costs of implementing process or product innovation due to the location of the firm or (more specifically) the innovation activity. For example, whether a firm does its engineering work onshore or offshore will change this cost of effort. We conceptualize this cost as the cost of engineering time needed to perform innovation activities. Note that in our paper we assume that the main difference between outsourcing or not is not in the cost of manufacturing itself (i.e., the firm could offshore manufacturing as well) but rather in the cost of innovation between the firm and its supplier.

The supply chain literature has traditionally focused on cost and supply chain control of the inventory decision (in addition to other decisions such as pricing, etc.) (e.g., Cachon 2003). Increasingly firms outsource to gain capabilities, not just reduce costs (McIvor 2008, PriceWaterhouseCoopers 2008). Empirical studies (e.g., Walker and Weber 1984, Argyres 1996) have shown that cost and capabilities are important drivers in deciding between vertical integration and outsourcing manufacturing. Here we characterize the specific trade-offs between cost and capability not only in manufacturing outsourcing, but also in the outsourcing of product and process innovation. Thus in this paper, we examine (in both deterministic and stochastic demand settings) these three drivers of control, capability, and cost together, where supply chain control in our context refers to the control of the supply chain innovation decisions.
We are interested in two research questions: First, when should a firm outsource process and product innovation, and second, what is the impact of outsourcing on the investments in innovation?

With regard to our first question of outsourcing innovation, we show that when the supply chain uses a wholesale pricing contract, the buyer outsources manufacturing/process innovation only when the supplier is much better in terms of capability and/or cost than the buyer. However, the buyer will never outsource both manufacturing and design (process and product innovation). On the other hand, when a two-part tariff contract is used, a sufficient condition for the buyer to outsource both manufacturing and design is that the supplier is better in one of the parameters (cost or process/product innovation capability) and equal in the other two. We also derive conditions under which outsourcing occurs if the two firms are dissimilar in their capabilities/costs.

With respect to our second question on the impact of outsourcing on the investment in innovation, we find the optimal innovation investment for each of the outsourcing scenarios. We show that in all three cases, the process innovation investment will be higher than the product innovation one, if the ratio between the demand and the margin (or a function of these for outsourcing manufacturing only) is higher than the ratio between the product and process innovation capabilities. Thus, products that are more functional in nature (Fisher 1997) will have, in general, higher effort investment in process innovation than in product innovation. We also define a Gap function which measures the capability differences between the two firms, and using this function, we provide conditions under which the investment in innovation will be higher when outsourcing. In particular, we show that if the firms are specialized such that the supplier dominates in process innovation and the buyer in product innovation, the investment in innovation may be higher when the supply chain has split control (where each firm controls one innovation decision) in the case of outsourcing manufacturing than that when producing in-house or outsourcing both manufacturing and design.

2. Literature Review

There is a vast literature on outsourcing manufacturing from various perspectives (e.g., Lee and Tang 1996, van Mieghem 1999, Baiman et al. 2001, Iyer et al. 2005, Ülkü et al. 2005, Amaral et al. 2006, Feng and Lu 2012a). As such, we discuss only the ones highly relevant to our model here. Bengtsson et al. (2009) recognize that firms may have different strategies in outsourcing, aiming at either low cost or innovation. In the supply chain literature, the use of supply chain contracts (such as wholesale-price, quantity-discount and two-part tariff) in outsourcing manufacturing has been studied extensively (e.g., Cachon 2003, Dong and Zhu 2007, Erhun et al. 2008) showing that some contract types (e.g., quantity-discount and two-part tariff) eliminate the issue of supply chain control. Further work on outsourcing manufacturing analyzes the impact of outsourcing on process innovation (cost reduction) as we do.
Bernstein and Kök (2009) model supplier cost reduction under different pricing contracts in a dynamic game, and find that investment in cost reduction is lower in a decentralized supply chain. Focusing on information leakage as a barrier to outsourcing, Gilbert et al. (2006) study competing OEMs (original equipment manufacturers) who decide whether or not to outsource, where either the OEM or the supplier chooses to invest to reduce unit cost, finding that outsourcing reduces the intensity of cost competition. Our study differs from the above in that we consider both process innovation and product innovation in a complete information setting without competition.

Outsourcing product innovation only, such as R&D outsourcing, has been modeled in papers including Huang et al. (2009) and Lai et al. (2009). Aghion and Tirole (1994) examine the impact of contracting on innovation effort when effort is unverifiable, finding that hold-up, a form of a transaction cost, may occur, resulting in underinvestment in innovation. In our model, since the firm outsources both manufacturing and innovation, the firms do not contract on the innovation efforts but rather on the payment terms for the units being purchased. Feng and Lu (2012b) study how design decisions differ and how product differentiation can be maintained when two competing OEMs outsource to a common ODM. Plambeck and Taylor (2005) examine outsourcing manufacturing and capacity decisions with two OEMs and a single contract manufacturer. They find that the OEMs do not invest as much in product innovation investments as in the non-outsourced case, unless they have high confidence in their bargaining power. In this paper that is based on a model without competition, we show a converse result when the buyer outsources manufacturing only: the process and product innovation efforts can be higher than in the centralized case, if the supplier is more capable or has a lower innovation cost.

Barney (1999) provides a conceptual overview of the role of capabilities in firm boundary decisions, including outsourcing. Gray et al. (2009) empirically study the impact of intended capabilities (i.e. competitive priorities) in cost and quality on the propensity to outsource. They find that a low cost priority leads to higher likelihood of outsourcing but that a quality priority does not systematically impact the propensity to outsource. Leiblein and Miller (2003) empirically demonstrate that increased capability in production leads to less outsourcing. Afuah (2000) empirically studies the role of supplier capabilities to the firm’s performance, when technology shifts render supplier capabilities obsolete. We study the impact of realized capabilities when suppliers have relevant capabilities.

Our model captures the propositions tested empirically by Argyres (1996) and Ulrich and Ellison (2005). Argyres (1996) argues that both transaction costs and capabilities play a role in outsourcing decisions, and specifically that the firm with the higher capability will perform the task. He finds support for his argument through a detailed case study of several design and production decisions in a single firm. We extend this insight by showing the capability being higher is not enough, but that the gap between the supplier’s and the buyer’s combination of capabilities and costs has to be high enough for the activity to
be outsourced due to the impact of supply chain control. Ulrich and Ellison (2005) identify the three outsourcing scenarios we analyze, and discuss the reasons for locating production and design in the same firm (referred to as integration). We show that if the supplier dominates in process innovation and the buyer in product innovation, split control (of the innovation activities) will result in higher innovation than outsourcing both process and product innovation when using a two-part tariff contract.

Others have also examined effort-based models. For example, Corbett and Decroix (2001) study shared savings contracts where both the supplier and buyer invest effort. In a service setting, Roels et al. (2010) study collaborative effort where outcomes depend on each firm’s nonverifiable effort. Recently, Li (2013) examined supplier effort in a competitive setting with multiple suppliers where renegotiation might occur after cost reducing effort was invested. Investing in product innovation effort is similar to improving quality. Zhu et al. (2007) investigate whether the supplier or buyer or both firms should invest in improved quality, finding the buyer often must invest in quality improvement even if manufacturing is outsourced. Kaya and Özçer (2009) study contracts for private quality costs and settings with noncontractible quality between an OEM who outsources both manufacturing and design, while the supplier invests in product quality. In a horizontal differentiation model, Wang and Shin (2012) examine product innovation leading to quality enhancement when a product is outsourced, focusing on the impact of downstream competition and contract type. They find innovation may increase with the buyer setting the wholesale price and may increase or decrease with bilateral bargaining.

Our analysis complements and extends these earlier works by looking at both process and product innovation, as well the outsourcing of manufacturing and possibly design in one model. Although portions of these questions have been addressed in the literature, in this paper we analyze a unified framework that captures the three factors of control, capability, and cost, and examines the incentives for both process and product innovation in a two-stage supply chain. Analyzing the three C’s together, along with product and process innovation, allows us to compare outsourcing manufacturing with outsourcing both design and manufacturing when the unit cost is not the only basis of the outsourcing decision.

3. The Model and Analysis

3.1. Model Description

Consider a firm supplying a product to end consumers. The firm, also called the buyer², must decide whether to retain control over manufacturing and design, outsource manufacturing (including process innovation), or outsource both manufacturing and design (both process and product innovation). We use a game theoretic approach, where the supplier is the Stackelberg leader. The buyer charges a fixed retail

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² We refer to this agent as the buyer, since although in the first case he manufactures, in the other two cases he outsources this function.
price per unit of \( p \) and each unit initially costs \( c_0 \) to manufacture. If outsourced, the supplier charges the buyer a wholesale price \( w \) per unit, and (if a two-part tariff contract is used) a fixed fee, \( F \). A two part tariff contract in this case can be seen as an abstraction of, for example, the growing practice in the auto industry of paying some amount upfront for engineering work and a per unit price upon production (Automotivesupplychain.org 2013). Both account for the production costs as well as the investment in process or product innovation effort. The buyer (or the supplier if the activity is outsourced) chooses the amount of effort to put into process innovation (that results in a cost reduction), \( e_1 \), and/or product innovation (that results in a value increase), \( e_2 \), at a cost of \( d_j e_i^2 \), where \( e_i \in [0, \infty) \), \( i = 1, 2 \), and \( j = B, S \), for either the buyer or supplier\(^3\). The cost per unit of innovation effort, \( d_j \), may differ for the buyer and supplier as discussed in the Introduction. The quadratic cost function implies diseconomies of scale for all types of effort.

Eisenhardt and Martin (2000) define capabilities as the “specific strategic and organizational processes like product development, alliancing, and strategic decision making that create value for firms within dynamic markets by manipulating resources into new value-creating strategies” (p. 1106). The resources in our model are the engineering/labor hours, and the capabilities are the effectiveness of the process and product innovation processes. We model capability as a parameter that moderates the effectiveness of the resources put into the innovation process and may differ by firm. The capability for cost reduction is \( \gamma_j \) and for product innovation is \( \delta_j \), \( j = B, S \), and \( \gamma_j, \delta_j \in (0, 1] \). Taking the cost reduction effort into account, the product’s unit cost is modeled as:

\[
c(e_1) = c_0 (1 - \gamma_j e_1) \geq c_{\min} > 0.
\]

Note that \( c(e_1) \) must be greater than some minimum cost, \( c_{\min} > 0 \), because of technological limits that would prevent a product cost from falling below a certain positive threshold. Note also that while many firms use outsourcing to reduce the manufacturing cost per unit, for example by outsourcing to a low cost country and thus reducing the labor cost and/or the material cost (e.g., Amaral et al. 2006), in our model when the firm outsources, we hold the initial unit cost, \( c_0 \), constant in order to isolate the effect of outsourcing process innovation. In the conclusion section we discuss the impact of relaxing this assumption assuming differing initial costs for the buyer and supplier. End consumer demand is deterministic and a function of the product innovation effort and capability such that \( D(e_2) = (A - k(1 - \delta_j e_2)p) \). Later, in \( \S 7 \) we extend the model to the case of stochastic demand. The parameter \( A \) denotes the market size, and \( k \) indicates the level of price sensitivity, where \( A - kp \geq 0 \). The parameter \( k \) can also be seen as a measure of willingness-to-pay for quality (Atasu and Souza 2013), and thus, product

\(^3\) We use innovation effort and innovation investment synonymously throughout the paper. In addition, we use process innovation and cost reduction interchangeably.
innovation effort increases that willingness-to-pay due to improved product quality and/or improved performance. While in some cases changes in process and product design can affect both the cost and the demand, for the sake of tractability and to focus on the first order effects, we assume in our model that the process innovation effort, $e_1$, affects the cost, while the product innovation effort, $e_2$, affects the demand. This is similar to the assumption in Bhaskaran and Krishan (2009) where the innovation that improves quality does not impact the marginal cost. We assume that the firms are profit maximizers, manufacturing capacity is unlimited, regardless of which firm manufactures, leadtime is negligible, and complete information is available to all parties. The timing of decisions is discussed below.

Our 3Cs model is summarized in Table 1. We study three cases as shown in the table. Case $I$ is that of a centralized firm, which has chosen to do everything In-house. Case $OM$ stands for Outsource Manufacturing, where only manufacturing, and hence process innovation, is outsourced. The last case analyzed is $ODM$ (Outsource Design and Manufacturing), where both manufacturing and design and consequently both process and product innovation are outsourced. Our model includes four decisions: the wholesale price, $w$, the fixed fee, $F$ (in the OM and ODM cases), and the process and product innovation efforts, $e_1$ and $e_2$, respectively. Considering the 3Cs of outsourcing innovation we study, Table 1 shows that while the control driver is ingrained in the cases analyzed, the capability and costs drivers are represented in the model through each firm’s capability and cost parameters. From a control perspective, the OM case has split control, while in the $I$ and ODM cases, one party controls both the innovation and the transfer pricing (if any) decisions. In case OM, split control of the innovation decisions occurs because $e_1$ is decided by the supplier and $e_2$ by the buyer, which in turn also impacts the quantity because the wholesale price $w$ impacts $e_2$ and the demand $D(e_2)$ increases in $e_2$. Recall that we assume that if the activity is outsourced, then the innovation in that activity is also outsourced. Accordingly, the costs of effort and capabilities for the innovation activity are those of the firm doing the activity, as summarized in Table 1. We expect these to vary between the firms in practice as gains from outsourcing are expected when there is a difference in input costs or production technology (Choi 2007). In this model, innovation labor cost and innovation capability lead to these gains.

<table>
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<tr>
<th>Case</th>
<th>Control</th>
<th>Capability</th>
<th>Cost</th>
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<tr>
<td></td>
<td>$w, F$</td>
<td>$e_1$</td>
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<td>$I$</td>
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<td>Buyer</td>
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<tr>
<td>$OM$</td>
<td>Supplier</td>
<td>Supplier</td>
<td>Buyer</td>
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<tr>
<td>$ODM$</td>
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Recall the example of Ford. Ford began in the in-house case, and later moved to outsourcing some portion of its component manufacturing. More recently, it began outsourcing both manufacturing and design of some subsystems (e.g., the electric drivetrain to Magna International). Thus, for each of its many subsystems, Ford could choose to perform both manufacturing and design internally (case I), to outsource manufacturing while maintaining design internally (case OM), or to outsource both design and manufacturing (case ODM). A firm deciding on an outsourcing strategy clearly has many issues to consider, however, in this paper, we propose one approach to look at the effect of control, capability, and cost on outsourcing and on the resulting process and product innovation efforts.

We analyze each of the cases \((n = I, OM, ODM)\) in §3.2-3.4 with the optimal outsourcing decisions shown in §3.5. In what follows, we refer to the buyer as “he” and the supplier as “she”. For the OM and ODM cases, we use two types of contracts: a wholesale pricing contract that is commonly used in practice (Lariviere and Porteus 2001) and a two-part tariff contract that is known to coordinate the supply chain decisions and thus eliminate the issue of supply chain control (Cachon and Harker 2002). In all cases, we assume firms can credibly commit, thus there is no renegotiation. Note that our focus is on outsourced production of a product designed in-house, and outsourced production and product innovation, not co-development by buyers and suppliers (e.g., Bhaskaran and Krishnan 2009, Kim and Netessine 2013). In addition, we do not consider the case where a firm outsources innovation only. Although this does occur (witness the rise of design houses such as IDEO and Frog Design), in many industries this is not the case. For example, Ulrich and Ellison (2005) did not find this case in their sample of bicycle manufacturers. Further, the contract structure in this case will not typically be based on a wholesale price per unit, but instead on a fixed price, cost-plus or time and effort basis (Carson 2007), and thus will not fit our model.

3.2. The In-house Case (No Outsourcing) - Case I

The firm (buyer) in case I determines the process and product innovation efforts and the order quantity simultaneously. The buyer solves his problem by maximizing expected profits that are equal to:

\[
\max_{e_1, e_2} \pi_I(e_1, e_2) = (p - c(e_1))(A - k(1 - \delta_B e_2)p) - d_B(e_1^2 + e_2^2)
\]

s.t. \(c(e_1) \geq c_{\text{min}}\)

\(e_1, e_2 \geq 0\)

Solving for the buyer’s optimal decisions in the integrated case, there are two possibilities depending on whether the process innovation effort is constrained or not, and in the next Theorem, we present the unconstrained optimal process innovation case, which we use for later analyses. The constrained case can be found in Appendix A. All proofs can be found in Appendix B. Before we state the Theorem, we define the following ratio that connects the buyer’s demand and margin.
Definition 1: The product type ratio, $T_p$, is defined as:

$$T_p \equiv \frac{(A - kp)}{(p - c_o)} \quad (3)$$

Fisher (1997) differentiates between functional and innovative products based on average demand, margin, and demand uncertainty. The $T_p$ ratio allows us to characterize this distinction between functional and innovative products using our model parameters. The ratio will be high when demand is high and margin low (for example, in the case of functional products such as basic food), and low when demand is low and margin high (such as in the case of innovative products like consumer electronics). This ratio will be utilized to help answer our two research questions regarding outsourcing and the investment in process and product innovation. Theorem 1 shows the optimal decisions for the in-house case.

Theorem 1: If

$$T_p \leq \frac{(c_0 - c_{min})(1 - a_{1B}a_{2B})}{a_{1B}(p - c_o)} - a_{2B}, \quad (4)$$

then

$$e_1^* = \frac{\gamma_Bc_0(p - c_o)}{2d_B(1 - a_{1B}a_{2B})}(a_{2B} + T_p), \quad e_2^* = \frac{kp\delta_B(p - c_o)}{2d_B(1 - a_{1B}a_{2B})}(1 + a_{1B}T_p) \quad (5)$$

and the buyer’s optimal profit is:

$$\pi_i^B = \pi_i^B(e_1^*, e_2^*) = \frac{(p - c_o)^2}{2(1 - a_{1B}a_{2B})}(a_{2B} + 2T_p + a_{1B}T_p^2) \quad (6)$$

where $a_{1B} = (\gamma_Bc_0)^2/(2d_B)$, and $a_{2B} = (kp\delta_B)^2/(2d_B)$. 

Note that the condition $c(e_1) \geq c_{min}$ is translated in Theorem 1 for a condition on the product type ratio, $T_p$ (Equation 4). Thus, the unconstrained solution is valid when the product type ratio is lower than the threshold in (4), while if the condition doesn’t hold, the optimal innovation efforts are given by the constrained problem in Appendix A. By Theorem 1, we see that the process innovation effort $e_1^*$ increases in the buyer’s process innovation capability $\gamma_B$, and the product innovation effort $e_2^*$ increases in the buyer’s product innovation capability $\delta_B$. As the buyer becomes more capable, he can spend less time and effort to get the same benefit. Conversely (and intuitively), as the buyer’s cost of innovation effort increases, the effort (both types) decreases. Thus in the 3C’s framework, we see that improved capability will increase innovation efforts, while increased costs will decrease them.

3.3. Outsource Manufacturing Only - Case OM

In case OM, the buyer outsources manufacturing to the supplier but still performs product innovation internally. Due to the decentralization, there is additionally a wholesale price $w$ per unit, and (if a two-part tariff contract is used) a fixed fee, $F$, determined by the supplier and paid by the buyer. The supplier incurs $c(e_1) = c_0(1 - \gamma_S e_1)$ for each unit produced and determines the cost reduction effort, $e_1$, while the buyer decides on the product innovation effort, $e_2$. Thus the supplier’s process innovation capability
\(y_S\) determines the cost reduction effectiveness, while the buyer's product innovation capability \(\delta_B\) determines the value enhancing effectiveness. The sequence of events is as follows: 1) the supplier maximizes expected profits by choosing \(e_1\) and the wholesale price \(w\) and possibly the fixed fee, \(F\), and 2) the buyer maximizes expected profits by choosing \(e_2\). Below we describe the problems of the buyer and supplier for the two-part tariff contract. Note that the problems under the wholesale pricing contract are a special case when the fixed fee \(F\) equals 0.

The buyer’s problem is to maximize expected profit, choosing \(e_2\), given \(w\) and \(F\):

\[
\max_{e_2} \pi^B_{OM}(e_2) = (p - w)(A - k(1 - \delta_B e_2)p) - d_B e_2^2 - F
\]

s.t. \(e_2 \geq 0\)

This objective function is concave in \(e_2\), and thus the solution for the optimal product innovation effort \(e_{OM}^*\) is given by the first order condition.

The supplier maximizes expected profits by choosing \(w\), \(F\) and \(e_1\), given the buyer’s decision. The supplier's problem is therefore:

\[
\max_{w,F,e_1} \pi^S_{OM} (w,F,e_1) = (w - c(e_1))\left(A - k \left(1 - \delta_B e_{OM}^*(w)\right)p\right) - d_S e_1^2 + F
\]

s.t. \(c(e_1) \geq c_{min}\)

\(w \leq p\)

\(e_1 \geq 0\)

3.4. Outsource Design and Manufacturing - Case ODM

The buyer in case ODM outsources both process and product innovation. Accordingly, in this case the buyer by deciding to outsource is (in a way) “selling the firm” to the supplier who determines the amount of effort to put into process and product innovation, \(e_1\) and \(e_2\), respectively, as well as the wholesale price, \(w\), and fixed fee, \(F\). The supplier decision on \(e_2\) determines the consumer demand, which in the deterministic case is equal to the order quantity, and thus the buyer has no decisions to make after deciding to outsource (which is why he is practically “selling the firm”). The sequence of events is 1) the supplier makes her decisions on \(e_1\), \(e_2\), \(w\) and \(F\) by maximizing expected profits, and 2) the supplier’s decisions determine the demand and the supplier and buyer’s expected profits. The supplier's problem under the two-part tariff contract (again, the wholesale pricing contract is a special case when \(F = 0\)) in case ODM is:

\[
\max_{w,F,e_1,e_2} \pi^S_{OM} (w,F,e_1,e_2) = (w - c(e_1))(A - k(1 - \delta_S e_2)p) - d_S (e_1^2 + e_2^2) + F
\]

s.t. \(c(e_1) \geq c_{min}\)

\(w \leq p\)

\(e_1,e_2 \geq 0\)
3.5. Optimal Solutions for the Outsourcing Cases

Solving for the buyer’s and supplier’s optimal decisions in the two outsourcing cases, we obtain the solutions in Theorem 2. Similar to case I, there are two possibilities depending on whether the process innovation effort is constrained or not, and in the next Theorem, we present the unconstrained optimal process innovation case, while the constrained case can be found in Appendix A.

Theorem 2:

The optimal solutions and profits for cases OM and ODM are the following:

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<tr>
<th>Condition</th>
<th>Case OM</th>
<th>Case ODM</th>
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<td>( c(e^<em>_i,n) \geq c^</em>_{\text{min}} )</td>
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<tr>
<td>( T_p \leq \frac{(c_2-c^*_{\text{min}})(2-a_1s^a_2b)}{a_1s^a_2} - a_2B )</td>
<td>( T_p \leq \frac{(c_2-c^*_{\text{min}})(1-a_1s^a_2)}{a_1s^a_2} - a_2S )</td>
<td>( T_p \leq \frac{(c_2-c^*_{\text{min}})(1-a_1s^a_2)}{a_1s^a_2} - a_2S )</td>
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<td>( \frac{1}{T_p} &gt; \frac{(1-a_1s^a_2)}{a_2B} )</td>
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<td>( \frac{1}{T_p} &gt; \frac{(1-a_1s^a_2)}{a_2B} )</td>
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<tr>
<td>( \omega^*_n )</td>
<td>( p )</td>
<td>( p )</td>
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<tr>
<td>( \frac{y_0c_0(p-c_0)}{2d_1(1-a_1s^a_2b)}(a_2B + T_p) )</td>
<td>( \frac{p}{2d_1(1-a_1s^a_2b)}(1 + a_1s^a_2T_p) )</td>
<td>( \frac{kp\delta^*_n(p-c_0)}{2d_3(1-a_1s^a_2s)}(a_2S + T_p) )</td>
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<tr>
<td>( \pi^*_n )</td>
<td>( 0 )</td>
<td>( \pi^*_n )</td>
</tr>
<tr>
<td>( \frac{(p-c_0)^2}{2(1-a_1s^a_2b)}(a_2B + 2T_p + \frac{\tau^*_n}{a_2B}) )</td>
<td>( \frac{(p-c_0)^2}{2(1-a_1s^a_2s)(1-a_1s^a_2b)}(a_2S + 2T_p + a_1s^a_2T_p^2) )</td>
<td>( \frac{(p-c_0)^2}{2(1-a_1s^a_2s)}(a_2S + 2T_p + a_1s^a_2T_p^2) - \pi^*_n )</td>
</tr>
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where \( a_1S = (y_0c_0)^2/(2d_1s) \), \( a_2S = (kp\delta^*_n)^2/(2d_3s) \), \( \pi^*_n \) and \( \pi^*_n \) are the optimal buyer and supplier profits for case \( n = \text{OM}, \text{ODM} \).

As can be seen in Theorem 2, when the supplier uses a wholesale pricing contract in the OM case, there are two possible subcases depending on if the supplier’s wholesale price, \( w^*_{\text{OM}} \), is lower or higher than the retail price, \( p \). In the latter case (since the wholesale price could never be higher than the retail price), we set \( w^*_{\text{OM}} = p \), resulting in zero profit for the buyer. The buyer also earns zero profit under the wholesale pricing contract in the ODM case. Note that when the supplier uses the two-part tariff, the wholesale price in the OM case is \( w^*_{\text{OM}} = c(e^*_i,\text{OM}) \), and the fixed fee is positive guaranteeing the buyer’s profit equals that of the integrated case, \( \pi^*_n \). This is standard in two-part tariff contracts where the
wholesale price is set equal to the unit cost and then the supplier gives the buyer his reservation profit and takes the rest of the supply chain profit using the fixed fee. In the ODM case, on the other hand, the wholesale price is $w_{ODM}^* = p$, while the fixed fee $F_{ODM}^* = -\pi_f^*$, and thus this is opposite to standard two-part tariff contracts. This occurs because the supplier’s profit expression is convex in $w$ so the supplier charges a wholesale price equal to the retail price, and then uses the fixed fee to pay back the buyer, guaranteeing his profit under the integrated case.

Looking at the optimal solutions for the outsourcing cases, we see that, similar to the integrated case and regardless of the contract used, the process innovation effort, $e_{1,n}^*$ ($n = OM, ODM$), increases in the supplier’s process innovation capability, $y_S$, and the product innovation effort, $e_{2,n}^*$ ($n = OM, ODM$), increases in the respective firm’s product innovation capability, $\delta_j$ ($j = B, S$), while both efforts decrease in the cost, $d_j$ ($j = B, S$) (recall that in the OM case, the product innovation effort depends on the buyer’s product innovation capability, while in the ODM case it depends on the supplier’s). In §5, we examine how the differences between the buyer and supplier capabilities and costs impact the investment in both process and product innovation.

4. To Outsource or Not to Outsource

In this section we examine our first research question: When will the buyer decide to outsource product and process innovation? We examine both the case of outsourcing manufacturing only (OM case) and the case where the buyer outsources both manufacturing and design (ODM case). In both cases the buyer will outsource when his profits are higher than in the centralized case (case I). We start by considering the wholesale pricing contract. Recall that as seen in Theorem 2 for the OM case, there are two possible subcases: If the wholesale price, $w_{OM}^*$, set by the supplier is lower than the retailer price, $p$, which occurs under the condition $1/T_p > (1 - a_{1S}a_{2B})/a_{2B}$, then the buyer outsources manufacturing under a certain condition. Specifically, the supplier outsources manufacturing when the product type ratio, $T_p$, is low, which corresponds to a more innovative product (Fisher 1997). Otherwise, $w_{OM}^* = p$, the buyer’s profit is zero, and thus he would choose not to outsource under the wholesale pricing contract. In the ODM case, because it is optimal for the supplier to set $w_{ODM}^* = p$, the buyer’s profit is always zero, and accordingly, he will never outsource under the wholesale pricing contract. This can be seen in the following Theorem:

**Theorem 3:** If the supplier is using a wholesale pricing contract, then
(a) If $1/T_p > (1 - a_{1S}a_{2B})/a_{2B}$, then the buyer outsources manufacturing if and only if
$$a_{1S}a_{2B} (4 - a_{1S}a_{2B}) \geq 3 + a_{1B}a_{2B}.$$  
(10)
Otherwise, the buyer never outsources manufacturing.
(b) The buyer never outsources both manufacturing and design.
By Theorems 1 and 2, it can be seen that for the optimal efforts to be positive, we need $a_{1j}a_{2j} < 1 \ (j = B, S)$ for cases I and ODM, while for case OM, we need $a_{1S}a_{2B} < 2$ when using the wholesale pricing contract and $a_{1S}a_{2B} < 1$ for the two-part tariff. When $a_{1S}a_{2B} < 2$ holds for the wholesale pricing contract, we see for Theorem 3(a) (case OM) that the left hand side (LHS) of the equation is increasing in $a_{1S}a_{2B}$, while the right hand side (RHS) is increasing in $a_{1B}a_{2B}$, and the maximum of both sides is close to 4. If $a_{1S}a_{2B} < 1$, then condition (10) in Theorem 3(a) never holds, while if $1 < a_{1S}a_{2B} < 2$, this condition might hold. Therefore for a given $a_{2B}$, the buyer will be more likely to outsource when the difference between $a_{1S}$ and $a_{1B}$ is large enough, which is dependent on the innovation cost and process capability of the two parties. It is interesting to note that in the OM case, due to the split control, the supplier has more of an incentive to reduce the wholesale price to get the buyer to invest in demand-increasing product innovation, and thus we find some conditions under which the buyer will outsource. On the other hand, in the ODM case, due to the full control of the innovation decisions by the supplier, the supplier doesn’t want to give any of the profits (i.e., rent) to the buyer, and therefore he will never outsource. Consequently, Theorem 3 shows that, similar to the extant supply chain literature, the wholesale pricing contract is very inefficient in both case OM and ODM in enabling the buyer to outsource innovation. Accordingly, in what follows we analyze the two-part tariff contract that is known in the supply chain literature for eliminating the issue of supply chain control. Before we discuss outsourcing under the two-part tariff contract, we formalize the conditions that guarantee the optimal efforts in Theorems 1 and 2 will be positive under a two-part tariff contract.

**Remark 1:** In the following analysis, we assume that $a_{1j}a_{2j} < 1 \ (j = B, S)$ and $a_{1S}a_{2B} < 1$, because without this assumption, the unconstrained optimal process innovation effort, $e_{*n} \ (n = I, OM, ODM)$, is negative. Although addressing this issue is straightforward, doing so will create cumbersome notation and additional conditions throughout.

Under the two-part tariff contract, the supplier (assuming her profit is positive) guarantees that the buyer will get his optimal profit under the in-house case, $\pi^B_i$. The next theorem shows when outsourcing occurs under this contract.

**Theorem 4:** If the supplier is using a two-part tariff contract, then
(a) The buyer outsources manufacturing if and only if $a_{1S} \geq a_{1B}$.
(b) The buyer outsources manufacturing and design if and only if

$$\frac{(a_{2S} + 2T_p + a_{1S}T_p^2)}{(1 - a_{1S}a_{2S})} > \frac{(a_{2B} + 2T_p + a_{1B}T_p^2)}{(1 - a_{1B}a_{2B})}$$

For the OM case in (a), we can see that if the supplier’s process capability is higher than the buyer’s capability (or her cost is lower), the supplier can induce the buyer to outsource by giving him his profit
under the in-house case (case I). This same condition ensures that the two-part tariff is feasible, otherwise the supplier’s profit would be negative. For case ODM in (b), note that the two sides are functionally identical where the LHS depends on the supplier’s parameters and the RHS on the buyer’s. This comes from the fact that as discussed in §3, the buyer is (practically) “selling the firm” to the supplier in case ODM. As seen in Theorems 1 and 2, the buyer’s and supplier’s profits are structurally identical in the three cases with a two-part tariff contract, and thus the buyer will outsource if the supplier has some advantage over him in capability or cost. By the definition of $a_{1j}$ and $a_{2j}$ ($j = B, S$), in Theorems 1 and 2, the outsourcing decision depends on the process innovation capability, $y_j$, the product innovation capability, $\delta_j$, and the innovation cost, $d_j$, of the two parties. This leads to the following corollary.

**Corollary 1**: If the supplier is using a two-part tariff for both OM and ODM, then:

(a) If the supplier’s costs and capabilities are all equal to those of the buyer, the total supply chain profits in case I, OM, and ODM are equal and thus outsourcing will never occur.

(b) Whenever two of the cost and capability parameters for the supplier and buyer are equal and the supplier dominates in the third parameter, outsourcing always occurs.

By Corollary 1, if for example, $y_S > y_B$, $\delta_S = \delta_B$, and $d_S = d_B$, then $a_{1S} > a_{1B}$ and $a_{2S} = a_{2B}$, which guarantees the condition for outsourcing for both OM and ODM. However, note that by Theorem 4, the buyer might still decide to outsource even when (for instance) the supplier’s cost is higher if his process and product capabilities are high enough to compensate for this higher cost.

5. **Impact of Outsourcing on the Innovation Investments**

In this section we study how the outsourcing decision affects the investment in innovation, by analytically comparing the investments in process and product innovation within and between the different outsourcing cases. Note that since we saw in the last section that under the wholesale pricing contract the buyer will usually not outsource (never in case ODM and only under some conditions in OM), we focus in this section on the two-part tariff contract. Furthermore, for ease of exposition and in order to focus on the main results, recall that we restrict our attention to the unconstrained process innovation effort case shown in Theorems 1 and 2.

5.1. **Process versus Product Innovation Within a Case**

We now determine under what conditions the investment in process innovation is greater than that in product innovation. As can be seen in the next theorem, $e^*_{1,n}$ may be higher or lower than $e^*_{2,n}$ ($n = I, OM, ODM$) depending on the firm’s capabilities and the product type ratio.

**Theorem 5**: If $c(e^*_{1,n}) \geq c_{min}$, for $n = I, OM, ODM$, then

a) $e^*_{1,l} > e^*_{2,l}$ if and only if
\[ T_p > \frac{k p \delta_B}{y_B c_0}. \]  

(11)

b) \( e_{1,OM}^* > e_{2,OM}^* \) if and only if

\[ \left( \frac{d_B}{d_S} \right) \left( a_{2B} + T_p \right) > \frac{k p \delta_B}{y_S c_0}. \]  

(12)

c) \( e_{1,ODM}^* > e_{2,ODM}^* \) if and only if

\[ T_p > \frac{k p \delta_S}{y_S c_0}. \]  

(13)

In Theorem 5(a) and (c), the LHS of the condition is the product type ratio, \( T_p \), while the RHS for case I (ODM) is the ratio of the buyer’s (supplier’s) capability in increasing demand through product innovation (per unit of effort) and his capability in decreasing cost (per unit of effort). Recall that in both cases I and ODM, there is full control of the innovation decisions so a single firm controls (decides) both types of innovation investment. When \( T_p \) is high (when demand is high and margin low as in the case of functional products), this condition will be easier to satisfy, while when \( T_p \) is low (when demand is low and margin high as in the case of innovative products), the condition will be harder to satisfy. These conditions might help explain Fisher’s (1997) assertion that industries that produce functional products should have efficient supply chains. By Theorem 5, this could be achieved through more investment in process innovation than product innovation (e.g., lean processes), while the opposite is true for industries that produce innovative products with responsive supply chains.

For case OM in Theorem 5(b), the right hand side of the condition is similar to those of cases I and ODM, taking into account who is making a particular decision (i.e., that the buyer invests in product innovation and the supplier in process innovation). Note that in case OM, unlike the other two cases, because different parties are making the investment decisions, there is split control, thus the left hand side is more complex than parts (a) and (c) but is still a function of the product type ratio, \( T_p \).

5.2. Process and Product Innovation Between Cases

We now turn to comparing the investments in process and product innovation between the three cases, examining the impact of outsourcing and control on the investment in innovation. Before comparing the different cases, we first define a function of the problem parameters that measures the effectiveness of the buyer and supplier with respect to two of the 3Cs, capability and cost (Recall that the third C, Control, is ingrained in the three cases we analyze).

**Definition 2**: Let \( G_{1j} = \frac{\gamma_j}{d_j (1-a_{1j} \alpha_{2j})} \) and \( G_{2j} = \frac{\delta_j}{d_j (1-a_{1j} \alpha_{2j})} \) be the capability-cost functions for \( j = B, S \), and let \( G_i = G_{iB}/G_{iS} \) (\( i = 1, 2 \)) be the Gap functions.
The *Gap functions* measure the ratio between the capabilities and costs of the supplier and buyer. $G_1$ represents the process capability-cost gap between the buyer and supplier, while $G_2$ similarly represents the product capability-cost gap. As the next lemma shows, the capability-cost functions are increasing in the respective firm’s capabilities and decreasing in her costs. Thus a higher capability-cost function indicates a higher effectiveness of the firm’s innovation investment.

**Lemma 1**: $G_{ij}$, $i = 1, 2$, are increasing in $y_j$ and $\delta_j$ and decreasing in $d_j, j = B, S$.

In the next theorem, we compare the three cases and examine in which of them the investing firm will devote more effort in each type of innovation.

**Theorem 6**: For $c(e_{ij, n}^*) \geq c_{\text{min}}, n = I, OM, ODM$, (a) If $G_1 > \left(\frac{1 - a_1 a_2}{1 - a_1 a_{2B}}\right)$, then $e_{1,1}^* > e_{1,0M}^*$, and if $G_2 > \left(\frac{1 + a_1 b T_p}{1 + a_1 b_2 B}\right)$, then $e_{2,1}^* > e_{2,0M}^*$. (b) If $G_1 > \left(\frac{a_{2S} + T_p}{a_{2B} + T_p}\right)$ then $e_{1,1}^* > e_{1,0DM}^*$, and if $G_2 > \left(\frac{1 + a_1 b T_p}{1 + a_1 b_2 B}\right)$, then $e_{2,1}^* > e_{2,0DM}^*$. (c) If $\left(\frac{1 - a_1 a_2}{1 - a_1 a_{2B}}\right) > \left(\frac{a_{2S} + T_p}{a_{2B} + T_p}\right)$, then $e_{1,0M}^* > e_{1,0DM}^*$, and if $G_2 > \left(\frac{1 + a_1 b T_p}{1 + a_1 b_2 B}\right)$, then $e_{2,0M}^* > e_{2,0DM}^*$.

As can be seen in the Theorem, when the ratio between the capability-cost function of the buyer and the supplier, represented by the Gap functions ($G_i, i = 1, 2$), is high as in parts (a) and (b), the innovation investment will be higher when not outsourcing (case I). When the ratio of the functions is low, on the other hand, the investment will be highest when outsourcing either just manufacturing or both manufacturing and design. For part (c), process innovation effort may be higher in either case, depending on the two firm’s capabilities, but product innovation effort will be higher under case OM than ODM when $G_2$ is high. Thus, the $G_i$ functions enable us to capture the gap between the capabilities and cost of the buyer and supplier and identify the threshold under which the investment in innovation will be higher when outsourcing.

As firms focus more on increasing innovation, searching for suppliers with sufficiently higher innovation capabilities and/or lower innovation cost becomes critical to their success. An interesting implication of Theorem 6 deals with the impact of control on the investment in innovation. In the next corollary, we show conditions under which case OM (where the innovation decisions are split between the firms) dominates cases I and ODM, where one firm has full control.

**Corollary 2**: Case OM (when the supply chain has split control) will have higher investment in innovation than cases I and ODM if

$$\max \left(1 - a_{15} a_{2S}, \frac{a_{2S} + T_p}{a_{2B} + T_p}\right) < \left(1 - a_1 b_2 B\right)$$ and $G_2 > \left(\frac{1 + a_1 b T_p}{1 + a_1 b_2 B}\right)$. (14)
By Corollary 2, when $G_1$ is low enough and $G_2$ high enough, innovation will be higher when there is split control. This occurs when the supplier has higher process innovation capability (or lower cost) but the buyer has superior product innovation capability. This means that outsourcing manufacturing (but not design) allows the two firms to specialize, focusing on their strengths. This is often seen in practice when firms such as Apple focus on product design, while their supplier (e.g., Foxconn) focuses on manufacturing (and process innovation) (Duhigg and Bradsher 2012).

6. Outsourcing and Investment in Innovation: A Numerical Example

Recall that we are interested in two main questions: First, when should a firm outsource manufacturing (and process innovation) versus both manufacturing and design (process and product innovation), and second, what is the impact of outsourcing on the investments in innovation? In §4, we determined conditions under which the buyer will outsource, while in §5, we found conditions comparing the investment in innovation within and between the cases. In this section, we seek to numerically illustrate these conditions and examine both questions by illustrating the impact of the 3Cs (control, capability, and cost) on the decision to outsource innovation and the resulting innovation investment.

Recognizing that supply chain control is ingrained in the scenario considered (I, OM, or ODM), we vary the capabilities and/or costs to determine the firm’s outsourcing decision in our analysis. For the illustration, we fix $p = 10, A = 2, k = 0.1, c_0 = 7, c_{min} = 0.1$. This allows us to compare across scenarios for a fixed initial deterministic demand level, $A - kp = 1$. The constant initial unit cost of 7 allows the impact of process and/or product innovation capability and the cost of innovation to be easily compared. We also fix the buyer’s process innovation capability and cost: $\gamma_B = 0.1, d_B = 1$.

We first explore the impact of process innovation capability, $\gamma_j$ ($j = B, S$), on the outsourcing decision. Figure 1 shows the lowest ratio of $\gamma_S$ to $\gamma_B$ which induces outsourcing by the buyer for a given cost ratio ($d_B/d_S$) for three possible cases: The OM case with a wholesale pricing contract (OM w), the OM case with a two-part tariff contract (OM 2PT) and the ODM case with a two-part tariff contract (ODM 2PT). (Recall that it is never optimal to outsource in case ODM with a wholesale pricing contract.) In the figure, we examine this threshold capability for two levels of product innovation capability (the same for buyer and supplier): part (a) with $\delta_B = \delta_S = 0.4$, and part (b) with $\delta_B = \delta_S = 0.95$. For example, if the cost of innovation for the buyer is half that of the supplier ($d_B/d_S = 0.5$), the supplier’s process innovation capability has to be at least 50% higher than the buyer’s ($\gamma_S/\gamma_B = 1.5$) for the buyer to want to outsource. Note that in part (a), the figure includes only the OM and ODM cases with the two-part tariff contracts because with a wholesale pricing contract in case OM, the buyer would never choose to outsource for any level of supplier process innovation capability, $\gamma_S$ (the condition in Theorem 3(a) never holds).
Figure 1: Process Capability Threshold above which Outsourcing Occurs for OM w, OM 2PT, and ODM 2PT with (a) $\delta_B = \delta_S = 0.4$, (b) $\delta_B = \delta_S = 0.95$.

Comparing OM 2PT with ODM 2PT for either level of $\delta_j (j = B, S)$, we see that when the costs are equal for the buyer and supplier ($d_B / d_S = 1$), the outsourcing threshold occurs at the same point for both cases, where the supplier capability is equal to that of the buyer ($\gamma_S / \gamma_B = 1$). For higher supplier cost ($d_B / d_S < 1$), the ODM 2PT case requires the supplier to have a higher process capability to outsource than case OM 2PT, while the opposite occurs for lower supplier cost. When the supplier’s cost is high, the buyer is better off not outsourcing design and manufacturing to lower process capability suppliers. Only when the capability is high enough, as shown by the thresholds in the figure, will the supply chain gains from the outsourcing agreement be large enough to make up for the higher supplier cost. (Recall that the two-part tariff contract enables the supply chain profits and supplier profits to be aligned by giving the buyer a constant profit equal to his profit from the integrated case.) Comparing part (a) to part (b), note that because in part (b) the buyer (and the supplier) are more capable in terms of product innovation, a higher process innovation capability level is required for the buyer to choose to outsource manufacturing and design when $d_B / d_S < 1$. However, when $d_B / d_S > 1$, the supplier’s cost is lower than the buyer’s and the supplier is better suited in terms of cost to perform both types of innovation, so a lower capability for process innovation is needed to induce outsourcing. Notice that the threshold for OM 2PT is the same for both levels of $\delta_j (j = B, S)$ shown. This is because in case OM there is split control and the supplier performs process innovation while the buyer performs product innovation and thus the change in product innovation capability is not impacting the process capability threshold needed to outsource. Also observe in (b) that the wholesale pricing contract requires that the supplier be much better than the buyer in process innovation capability, compared to either two-part tariff contracts. This is due to the inefficiency of the wholesale price contract: The supply chain profits fall in the decentralized case, yet in order to outsource the buyer must earn higher profits than in the centralized case, necessitating the high $\gamma_S$. 
Figure 2 compares each firm’s profits and the total effort \( (e_{T,n} = e_{1,n} + e_{2,n}) \), \( n = I, OM, 2PT, ODM \), as supplier innovation cost changes. We do not show the OM case with a wholesale pricing contract because the buyer would never choose to outsource using such a contract as the condition in Theorem 3(a) does not hold. In (a), we see that the supplier’s profits are negative when \( d_B / d_S < 1 \), and thus the supplier would not participate for this range of cost where the supplier’s cost is higher than the buyer’s. Therefore, for \( d_B / d_S < 1 \), the buyer would choose the in-house case, and for \( d_B / d_S > 1 \), the buyer is indifferent (due to the 2PT contract) but the supplier (and the supply chain as a whole) would prefer case ODM. Note also that the buyer’s share of the supply chain profits is higher for almost all values of the supplier cost. Only when the supplier cost is close to half that of the buyer \( (d_B / d_S = 2) \), the supplier share of the supply chain profits is higher than 50%. Looking at part (b), when the buyer does not outsource \( (d_B / d_S < 1) \), the total effort is highest in case I and lowest in case ODM \( (e_{T,I} > e_{T,OM} > e_{T,ODM}) \). Looking at Theorem 6 for this case, we can see that all the conditions in Theorem 6 hold. When outsourcing might occur \( (d_B / d_S > 1) \), the opposite occurs \( (e_{T,ODM} > e_{T,OM} > e_{T,I}) \). This is because none of the conditions in parts (a)-(c) of Theorem 6 hold when \( d_B / d_S > 1 \).

Figure 2: (a) Buyer and Supplier Profits and (b) Total Effort \( (e_{1,n} + e_{2,n}) \), \( n = I, OM, 2PT, ODM 2PT, \) as Supplier Cost Changes for \( \gamma_S = 0.1, \delta_B = \delta_S = 0.95 \).  

Figure 3 is similar to Figure 2, except now the innovation cost is constant and the supplier process innovation capability changes. Similar to the example in Figure 2, the buyer never chooses to outsource with a wholesale pricing contract. Part (a) shows the buyer’s and supplier’s profits. The supplier’s 2PT profits are negative when \( \gamma_S / \gamma_B < 1 \), so the supplier would not participate with a 2PT contract, and the buyer would not choose a wholesale pricing contract. Thus for \( \gamma_S / \gamma_B < 1 \), the buyer chooses case I, and when \( \gamma_S / \gamma_B > 1 \), the two-part tariff contract will be used if the buyer chooses to outsource. The supplier is indifferent between the OM or ODM 2PT contract, which differs from the results in Figure 2. In both case OM and ODM, the supplier performs the process innovation, thus benefits similarly from the increasing capability shown in Figure 3. When instead costs differ as in Figure 2, the fixed fee the
supplier pays to the buyer in the 2PT contract is also different due to these costs. Similar to Figure 2, we can see in Figure 3 that only when the supplier’s capability is high \( \gamma_S / \gamma_B > 2.1 \) do her profits become higher than the buyer’s. Part (b) shows the total effort. When \( \gamma_S / \gamma_B < 1 \), no outsourcing occurs and the in-house case has the highest total innovation effort (Theorem 6(a) and (b) hold). For \( \gamma_S / \gamma_B > 1 \), the effort in case I is the lowest. The change in slope to negative in the OM 2PT and ODM 2PT effort levels occurs when \( c(e_1) < c_{min} \), and thus the results in Appendix A apply.

\[ \text{Figure 3: (a) Buyer and Supplier Profits and (b) Total Effort} \]

\[ (e_{1,n} + e_{2,n}), n = 1, \text{OM 2PT, ODM 2PT} \]

7. Extension: The Impact of Uncertainty

7.1. Model Description

Until now we have assumed that the demand faced by the buyer is deterministic. An interesting question is how the results of the model change when the demand is stochastic. The majority of the model setup in §3 remains unchanged. However, there are two main changes: First, in the three cases considered (I, OM, and ODM), the buyer now determines the quantity to produce/order, \( y \), in addition to the innovation effort decisions (that may be decided either by the buyer or the supplier). Second, the end consumer demand is now stochastic (while still a function of the product innovation capability and effort) such that \( D(e_2, \varepsilon) = (A - k(1 - \delta_2 e_2)p) + \varepsilon \) where \( \varepsilon \sim F(\cdot) \) on \([\alpha, \beta]\), with density \( f(\cdot) \), and \( E(\varepsilon) = 0 \). In order to have positive demand for all effort levels, we assume that \( A - kp + \alpha \geq 0 \). In our analysis, we use \( z \), the stocking factor, instead of \( y \) (similar to Petruzzi and Dada 1999), where \( y = (A - k(1 - \delta_2 e_2)p) + z \). The stocking factor represents the portion of stochastic demand \( \varepsilon \) satisfied and \( F(z) \) represents the fractile. We use the convention that \( \bar{F}(z) = 1 - F(z) \), where \( F(\cdot) \) can be any reasonable demand distribution. We also define the expected shortages as \( s(z) = \int_{\alpha}^{\beta} (\varepsilon - z) f(\varepsilon) d\varepsilon \). Throughout this section we use the hat (\( \hat{\cdot} \)) notation to indicate the stochastic case solutions.
The analysis of the three cases (I, OM and ODM) follows the same steps as in §3, with the additional decision variable, z. The buyer’s problem for the in-house (I) case is thus:

\[
\begin{align*}
\max_{z,e_1,e_2} & \quad \hat{\pi}_I(z, e_1, e_2) = -c(e_1)(A - k(1 - \delta_B e_2)p + z) + p(A - k(1 - \delta_B e_2)p - s(z)) - d_B(e_1^2 + e_2^2) \\
\text{s.t.} & \quad c(e_1) \geq c_{\text{min}} \\
& \quad \alpha \leq z \leq \beta \\
& \quad e_1, e_2 \geq 0.
\end{align*}
\]

In case OM, the buyer outsources manufacturing (and thus process innovation) to the supplier but still performs product innovation internally. The sequence of events is as follows: 1) the supplier maximizes expected profits by choosing \(e_1\), the wholesale price \(w\), and the fixed fee, \(F\) (when the two-part tariff is used), and 2) the buyer maximizes expected profits by choosing \(e_2\) and the stocking factor \(z\). Below we describe the problems of the buyer and supplier for the two-part tariff contract. Note that the problem under the wholesale pricing contract is a special case when the fixed fee, \(F\), equals 0. Thus the buyer’s problem is:

\[
\begin{align*}
\max_{z,e_2} & \quad \hat{\pi}_{OM}^B (z, e_2) = -w(A - k(1 - \delta_B e_2)p + z) + p(A - k(1 - \delta_B e_2)p - s(z)) - d_B e_2^2 - F \\
\text{s.t.} & \quad \alpha \leq z \leq \beta \\
& \quad e_2 \geq 0
\end{align*}
\]

This objective function is jointly concave in \(z\) and \(e_2\). Accordingly, the solutions for the optimal stocking factor \(\hat{z}_{OM}^*\) and optimal product innovation effort \(\hat{e}_{2,OM}^*\) are given by the first order conditions.

The supplier maximizes expected profits by choosing \(w\), \(F\), and \(e_1\), given the buyer’s decisions. Because the buyer chooses \(w = p\hat{F}(z)\), this is parallel to deciding on \(z, F\), and \(e_1\), and we can use the change of variable between \(w\) and \(z\) as in Lariviere and Porteus (2001). The supplier's problem is thus:

\[
\begin{align*}
\max_{z,F,e_1} & \quad \hat{\pi}_{OM}^S (z, F, e_1) = (p\hat{F}(z) - c(e_1)) \left( A - k \left( 1 - \delta_B \hat{e}_{2,OM}^* (z) \right) p + z \right) - d_S e_1^2 + F \\
\text{s.t.} & \quad c(e_1) \geq c_{\text{min}} \\
& \quad \alpha \leq z \leq \beta \\
& \quad e_1 \geq 0
\end{align*}
\]

In the ODM case, the buyer outsources both process and product innovation. The sequence of events is 1) the supplier makes her decisions on \(e_1, e_2, w\) and \(F\) by maximizing expected profits, and 2) the buyer maximizes expected profits to determine \(z\), knowing the wholesale price, fixed fee and efforts. Note again that the problem under the wholesale pricing contract is a special case when the fixed fee, \(F\), equals 0. The buyer’s problem is the standard decentralized newsvendor problem:

\[
\begin{align*}
\max_{z} & \quad \hat{\pi}_{ODM}^B (z) = -w(A - k(1 - \delta_S e_2)p + z) + p(A - k(1 - \delta_S e_2)p - s(z)) - F \\
\text{s.t.} & \quad \alpha \leq z \leq \beta
\end{align*}
\]

\[22\]
This is concave in $z$, and the buyer's optimal decision is the known critical fractile solution. The supplier determines both efforts and the wholesale price, given the buyer’s choice of $z$. Similar to the OM case, we use the change of variable between $w$ and $z$ and obtain the following supplier's problem:

$$
\max_{z,F,p,e_1,e_2} \tilde{H}_{ODM}^2 (z,F,e_1,e_2) = (pF(z) - c(e_1))(A - k(1 - \delta s e_2)p + z) + F - d_s(e_1^2 + e_2^2)
$$

\text{s.t.}
\begin{align*}
&c(e_1) \geq c_{\min} \\
&\alpha \leq z \leq \beta \\
&e_1, e_2 \geq 0
\end{align*}

### 7.2. Results

Solving for the buyer’s and supplier’s optimal decisions in each case above, we obtain the optimal solutions for the cases. Similar to §3, there are two possibilities depending on whether the process innovation effort is constrained or not, and in the next Theorem, we present the unconstrained optimal process innovation case, while the constrained case can be found in Appendix A. Before we state the Theorem, we define, similar to the deterministic case, the product type ratios that connect the buyer’s or supplier’s demand and margin for each of the cases.

**Definition 3**: The product type ratios, $T_{p,n}$, for case $n = I, OM, ODM$, are defined as:

$$
T_{p,I} \equiv \frac{(A - kp + \hat{z}_I)}{(p - c_0)}, \quad T_{p,OM} \equiv \frac{(A - kp + \hat{z}_{OM})}{(p - \hat{w}_{OM}^2)}, \quad T_{p,ODM} \equiv \frac{(A - kp + \hat{z}_{ODM})}{(\hat{w}_{ODM} - c_0)}. \quad (20)
$$

These ratios now vary by case because they include the optimal stocking factor for each case. Due to the complexity of the solutions, the optimal stocking factor cannot be found in closed form.

**Theorem 7**

The optimal solutions for the model with uncertainty for each of the cases are the following:

<table>
<thead>
<tr>
<th>Case</th>
<th>Case OM</th>
<th>Case ODM</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $c(\hat{t}<em>1,n) \geq c</em>{\min}$</td>
<td>$a_{1B} \leq \frac{c_0 - c_{\min}}{a_{2B}(p - c_0) + (A - kp + \hat{z}_I)}$</td>
<td>$a_{1S} \leq \frac{c_0 - c_{\min}}{a_{2B}p(\hat{t}<em>{OM}) + (A - kp + \hat{z}</em>{ODM})}$</td>
</tr>
<tr>
<td>Hessian</td>
<td>$a_{1B} &lt; h_I$</td>
<td>$a_{1S} &lt; h_{ODM}$</td>
</tr>
<tr>
<td>$\hat{e}_{1n}^*$</td>
<td>$\gamma_s c_0(p - c_0) \gamma_s (\hat{t}<em>{OM}) (a</em>{2B} + T_{p,I})$</td>
<td>$\gamma_s c_0(p - \hat{w}<em>{OM}^2) (a</em>{2B} + T_{p,OM})$</td>
</tr>
<tr>
<td>$\hat{e}_{2n}^*$</td>
<td>$\frac{kp \delta_s (p - c_0)}{2d_s(1 - a_{1B}a_{2B})} (1 + a_{1B}T_{p,I})$</td>
<td>$\frac{kp \delta_s (p - \hat{w}<em>{OM}^2)}{2d_s} (1 + a</em>{1S}T_{p,OM})$</td>
</tr>
<tr>
<td>$\hat{w}_{n}^*$</td>
<td>$pF(\hat{z}_{OM})$</td>
<td>$pF(\hat{z}_{ODM})$</td>
</tr>
<tr>
<td>$\hat{z}_{n}$ solves</td>
<td>$\frac{(1 - a_{1B}a_{2B})pF(\hat{z})}{(p - c_0)} = (1 + a_{1B}T_{p,I}) \frac{pF(\hat{z}<em>{OM}) - c_0}{pF(\hat{t}</em>{OM})} = a_{1S} \left( a_{2B} + T_{p,OM} \right)$</td>
<td>$\frac{(1 - a_{2S}pF(\hat{z}<em>{ODM}))}{(pF(\hat{z}</em>{ODM}) - a_{1S})T_{p,ODM}}$</td>
</tr>
</tbody>
</table>
wholesale pricing contract in the stochastic case, the wholesale price, stocking factor chosen by the buyer. Second, unlike the deterministic case, when the supplier uses a two-part tariff contract, then efforts are structurally similar, although in the stochastic case, the efforts are also impacted by the optimal stocking factor, 

\[ \frac{1}{2} \left( \frac{c_0}{a_{2B}} + \frac{c_0}{a_{2B}} \right) \]  

and hence we focus on the comparison of the innovation efforts between the stochastic and deterministic cases in order to assess the impact of uncertainty on the model. In the next subsection, we analytically extend for the stochastic model by defining stochastic gap functions that capture the difference in cost and capabilities between the supplier and the buyer. However, the results are not structurally different, and hence we focus on the comparison of the innovation efforts between the stochastic and deterministic cases in order to assess the impact of uncertainty on the model. In the next subsection, we analytically

### Table: Case OM vs. Case ODM

<table>
<thead>
<tr>
<th>Case OM</th>
<th>Case ODM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c(\hat{e}<em>{1,n}) \geq c</em>{min} )</td>
<td>( a_{1S} \leq \frac{c_0 - c_{min}}{a_{2B}p + (A - kP + \hat{z}_{OM})} )</td>
</tr>
<tr>
<td>Hessian</td>
<td>( a_{1S} &lt; \hat{h}_{OM} )</td>
</tr>
<tr>
<td>( \hat{c}_{1,n} )</td>
<td>( \frac{c_0}{2d_s} )</td>
</tr>
<tr>
<td>( \hat{c}_{2,n} )</td>
<td>( \frac{kp\delta_2(p - \hat{w}_{OM})}{2d_s} )</td>
</tr>
<tr>
<td>( \hat{w}_{OM} )</td>
<td>( p\hat{F}(\hat{z}_{OM}) )</td>
</tr>
<tr>
<td>( \hat{p}_{OM} )</td>
<td>( \hat{p}_{ODM} )</td>
</tr>
<tr>
<td>( \hat{z}_{OM} )</td>
<td>( (p\hat{F}(\hat{z}<em>{OM}) - c_0) + a</em>{1S}(a_{2B}p + (A - kP + \hat{z}_{OM})) )</td>
</tr>
</tbody>
</table>

Comparing the results of the stochastic model in Theorem 7 to those of the deterministic one in Theorems 1 and 2, three observations are evident: First, the optimal process and product innovation efforts are structurally similar, although in the stochastic case, the efforts are also impacted by the optimal stocking factor chosen by the buyer. Second, unlike the deterministic case, when the supplier uses a wholesale pricing contract in the stochastic case, the wholesale price, \( \hat{w}_n (n = I, OM, ODM) \), is always lower than the retail price, \( p \), and thus this contract can still result in outsourcing even in the ODM case. Lastly, note that in the stochastic case of Theorem 7, we do not have a closed form solution for the optimal stocking factor, \( \hat{z}^*_n \), and since the innovation efforts, \( \hat{e}_{in}(i = 1, 2, \text{and } n = I, OM, ODM) \), are a function of \( \hat{z}^*_n \), we do not have closed form solutions of the optimal profits and hence can evaluate the outsourcing question only numerically. With regard to the second research question of the investment in innovation, note that the results achieved for the deterministic case in §5 (Theorems 5 and 6) can be extended for the stochastic model by defining stochastic gap functions that capture the difference in cost and capabilities between the supplier and the buyer. However, the results are not structurally different, and hence we focus on the comparison of the innovation efforts between the stochastic and deterministic cases in order to assess the impact of uncertainty on the model. In the next subsection, we analytically
compare the innovation efforts between the deterministic and stochastic models, while in §7.4 we numerically explore the outsourcing question and how it changes with stochastic demand.

7.3. The Impact of Uncertainty on the Investment in Innovation

We now examine the impact of uncertainty on the optimal innovation efforts in each of the three cases (I, OM, and ODM). As we saw in §4, in the deterministic case under the wholesale pricing contract, the supplier will never outsource in case ODM and only rarely under case OM. Thus, in order to make the comparison on even terms, when examining the impact of uncertainty we assume that the supplier uses the two-part tariff contract. The next Theorem compares the innovation efforts of the deterministic case (in Theorems 1 and 2) with that of the stochastic case in Theorem 7.

**Theorem 8:** If the supplier is using a two-part tariff contract, \( c(\hat{e}_{1,n}^*) \geq c_{\text{min}}, \) and \( c(e_{1,n}^*) \geq c_{\text{min}}, \) then

(a) \( \hat{e}_{1,n}^* \geq e_{1,n}^* \) if and only if \( \hat{e}_{1,n}^* \geq 0 \) for \( n = I, ODM. \)

(b)(i) \( \hat{e}_{1,OM}^* \geq e_{1,OM}^* \) if and only if

\[
\hat{z}_{OM}^* \geq \left( \frac{1}{1-a_{15}a_{2B}} - \frac{p-w_{OM}}{p-c_0} \right) + a_{15}T_p (p-c_0)a_{2B} > 0. \tag{21}
\]

(ii) \( \hat{e}_{2,OM}^* \geq e_{2,OM}^* \) if and only if

\[
F(\hat{z}_{OM}^*) \geq \frac{(p-c_0)(1+a_{15}T_p)}{p(1-a_{15}a_{2B})} \tag{22}
\]

By Theorem 8, we can see that for the two cases that have full control (cases I and ODM) the impact of uncertainty is such that if the stocking factor is positive (i.e., the order quantity is higher than the mean), then uncertainty increases the investment in both process and product innovation and thus also the total innovation investment. For the OM case, since the RHS of (21) and (22) are positive (for (21) note that \( c_0 \leq \hat{w}_{OM} \) and \( a_{15}a_{2B} < 1 \)), the process and product innovation efforts under uncertainty will be higher than those under the deterministic case only for fractiles (i.e., stocking factor) that are above the thresholds given in (21) and (22). Thus for uncertainty to have a positive impact on the process innovation investment, the stocking factor for case OM needs to be higher than that for cases I and ODM due to the higher threshold.

7.4. Numerical Study: The Impact of Uncertainty

Using the same parameters as in §6, we examine numerically the impact of uncertainty on the supplier’s process innovation capability threshold needed to induce the buyer to outsource. Figure 4(a) shows this threshold. The level of process capability needed is highest for the two wholesale pricing contracts. Recall that control is split in case OM, with the buyer making the product innovation decision and the supplier making the process innovation decision. However, there is an additional aspect of supply chain control at work with stochastic demand and a wholesale pricing contract. When the demand is stochastic and the
supply chain uses a wholesale pricing contract, the split control (in case OM) results in both a lower innovation investment (for a given capability level) and a lower order quantity than optimal (as per the standard supply chain literature result). When the supplier’s innovation cost is high \( (d_B/d_S < 1) \), ODM w requires an even higher process capability because the supplier is performing both types of innovation at this higher cost. Similar to the deterministic case in Figure 1, we see that when the costs are equal for the buyer and supplier \( (d_B/d_S = 1) \), the outsourcing threshold occurs at the same point for both 2PT contracts, where the supplier capability is equal to that of the buyer \( (\gamma_S/\gamma_B = 1) \). Also similar to the Figure 1, when the supplier has higher cost \( (d_B/d_S < 1) \), ODM 2PT requires the supplier to have a higher process capability than OM 2PT, and for lower supplier cost, OM 2PT requires the higher process capability.

Figure 4(b) directly compares the ODM 2PT case for deterministic and stochastic demand. Interestingly, the process capability threshold needed to induce outsourcing is higher for stochastic demand when the supplier has higher costs, but lower when the supplier has lower costs. This indicates that with stochastic demand, outsourcing manufacturing and design to a low innovation cost supplier requires a relatively lower supplier process innovation capability (all else equal). Note however, that as the supplier becomes much lower cost \( (d_B/d_S > 1.5) \), the difference between the stochastic and deterministic cases disappears and the cost benefit dominates.

**Figure 4:** (a) Supplier Process Capability Threshold above which Outsourcing Occurs in OM w, OM 2PT, ODM w, and ODM 2PT, (b) Comparison of ODM 2PT Supplier Process Capability Threshold in Deterministic and Stochastic Cases. Parameters used: \( \delta_B = \delta_S = 0.4 \)

### 8. Conclusions

Outsourcing continues to be of interest to both firms and academics. In this paper, we model three specific aspects of outsourcing process and product innovation that we call the 3Cs of outsourcing innovation: Control (of both innovation and inventory/ordering decisions), firm innovation Capabilities, and the Cost of innovation. We find that these three factors have highly interrelated effects on our two key questions: When should a firm outsource manufacturing (process innovation) versus manufacturing...
and design (process and product innovation) and what is the impact of outsourcing on the investment of innovation effort?

With regard to our first question of outsourcing, we show that when the supply chain uses a wholesale pricing contract, the buyer outsources manufacturing when the supplier is much better in terms of capability and/or cost than the buyer; however, the buyer will never outsource manufacturing and design. This is in line with the insight of Jacobides (2004) who found that if capabilities are evenly distributed at an industry level, no trade between firms will occur. We confirm this at the firm level. When a two-part tariff contract is used in our analysis, the buyer will outsource both manufacturing and design if the supplier dominates him in cost or process/product innovation capability. With respect to the second question, we show that for all outsourcing scenarios (I, OM and ODM), a firm may choose innovation effort investment strategies based on the type of product, functional or innovative (Fisher 1997). Functional products, due to larger markets and smaller margins, are more likely to have higher process innovation efforts than product innovation efforts, while the opposite is true for innovative products. In addition, we define the *Gap function* between firm capabilities and, using that, provide conditions under which the investment in innovation will be higher when outsourcing. We also prove that when the firm chooses to outsource using a two-part tariff, a higher total innovation investment often occurs than when produced in-house. Furthermore, our analysis reveals that, even when the supply chain has split control, the supply chain may still invest more in innovation effort (compared to producing in-house or outsourcing both manufacturing and design) if the firms are specialized, where the supplier dominates in process innovation capability and the buyer in product innovation capability. This supports the common practice of outsourcing to firms that specialize in manufacturing, such as contract manufacturers like Foxconn.

Finally, we extend our model to the case of stochastic demand examining the impact of uncertainty on the outsourcing decision and the investment in innovation. We illustrate that when outsourcing manufacturing and design (case ODM), uncertainty requires the supplier to have a higher process innovation capability advantage (compared to the deterministic case) when her cost is higher than the buyer’s and a lower capability advantage when it is lower. In other words, if the supplier has lower costs of innovation, demand uncertainty increases outsourcing (all else equal). With respect to the innovation investment question, we show that when the supply chain has full control (cases I and ODM), an optimal order quantity higher than the mean (in the stochastic case) implies that uncertainty increases the investment in both process and product innovation and thus also the total innovation investment. On the other hand, when control is split (case OM), the order quantity needs to be significantly higher than the mean to guarantee a higher investment in innovation under uncertainty. Using newsvendor logic this implies that when there is control (Cases I and ODM), in industries where the understocking costs are
slightly higher than the overstocking costs, uncertainty will cause a higher innovation investment, while when control is split (in the OM case), only industries where the understocking costs are much higher than the overstocking costs will have higher innovation investment due to uncertainty.

Our analysis shows the importance of assessing buyer and supplier relative capabilities, in addition to costs. Firms do not necessarily have this knowledge (Doig et al. 2001), but undertaking initiatives such as balanced sourcing (Laseter 1998) to more thoroughly assess the potential supply base before determining potential suppliers or renegotiating contracts is essential.

The paper could be extended in several directions. A relatively straightforward extension is to consider the impact of a different (possibly lower) initial cost $c_0$ for the supplier versus the buyer to account for the phenomena of outsourcing to gain a manufacturing cost advantage. Many of the results would still hold, albeit with different conditions. Clearly this would make outsourcing of process and/or product innovation along with manufacturing more appealing, but would make the impact of outsourcing design in addition to manufacturing less apparent. Another direction to explore is the impact of competition, either between buyers or suppliers (e.g., Cachon and Harker 2002, Feng and Lu 2012a). With the inclusion of both process and product innovation effort decisions, the effect of competition is not obvious. Competition can also arise between the supplier and the buyer, when as a result of the outsourcing, the supplier becomes a competitor. For example, Samsung, a supplier to Apple, released its own version of the smartphone, the Galaxy phone that is now the biggest competitor to the Apple iPhone (Seitz 2012). Furthermore, our model is static. It does not address the future value of capabilities, which is a key argument against outsourcing (Pisano and Shih 2012). For example, learning curves imply reduction in cost over time, however, the buyer may have a faster rate of learning than the supplier, so even if the buyer starts at a cost disadvantage, it may improve faster (Berggren and Bengtsson 2004, Gray et al. 2009). Alternatively, outsourcing might cause the buyer to lose capabilities over time, affecting future products and profits, potentially due to loss of learning opportunities (Anderson and Parker 2002).

Outsourcing continues to be an important business phenomena, and our model is an attempt to better understand one critical part, outsourcing process and product innovation.

References


Appendix A

Theorem A1: Deterministic Demand Constrained Case

The optimal solutions for the deterministic case when \( c(e^*_1) \) is constrained and a wholesale pricing contract is used are as follows:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Case I</th>
<th>Case OM</th>
<th>Case ODM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c(e^*<em>1) &lt; c</em>{min} )</td>
<td>( T_p &gt; \frac{(c_o - c_{min})(1-a_{1S}a_{2B})}{a_{1B}(p-c_o)} - a_{2B} )</td>
<td>( T_p &gt; \frac{(c_o - c_{min})(1-a_{1S}a_{2B})}{a_{1S}(p-c_o)} - a_{2B} )</td>
<td>( T_p &gt; \frac{(c_o - c_{min})(1-a_{1S}a_{2S})}{a_{1S}(p-c_o)} - a_{2S} )</td>
</tr>
<tr>
<td>( w &lt; p )</td>
<td>-</td>
<td>( \frac{1}{T_p} \geq \frac{(1-a_{1S}a_{2B})}{a_{2B}} )</td>
<td>-</td>
</tr>
<tr>
<td>( e^*_1 )</td>
<td>( \frac{c_0 - c_{min}}{\gamma Y c_0} )</td>
<td>( \frac{c_0 - c_{min}}{\gamma Y c_0} )</td>
<td>( \frac{c_0 - c_{min}}{\gamma Y c_0} )</td>
</tr>
<tr>
<td>( e^*_2 )</td>
<td>( \frac{k p \delta_B}{2d_B} (p - c_{min}) )</td>
<td>( \frac{a_{2B}(p - c_{min}) - (A - k p)}{2k p \delta_B} )</td>
<td>0</td>
</tr>
<tr>
<td>( w^*_n )</td>
<td>-</td>
<td>( \frac{A - k p + a_{2B}(p + c_{min})}{2a_{2B}} )</td>
<td>( p )</td>
</tr>
<tr>
<td>( \pi^B_n )</td>
<td>( \frac{(p - c_{min})(A - k p)}{8a_{2B}} + \frac{a_{2B}}{2} (p - c_{min})^2 )</td>
<td>( \frac{1}{8a_{2B}} (2a_{2B}(p - c_{min})(A - k p) - 3(A - k p)^2 + a_{2B}^2(p - c_{min})^2) )</td>
<td>0</td>
</tr>
<tr>
<td>( \pi^S_n )</td>
<td>-</td>
<td>( \frac{1}{8a_{2B}} (a_{2B}(p - c_{min}) + (A - k p)^2 - \frac{(c_0 - c_{min})^2}{2a_{1S}}) )</td>
<td>( \frac{(p - c_{min})(A - k p) + a_{2S}(p - c_{min})^2 - (c_0 - c_{min})^2/2a_{1S}}{2a_{1S}} )</td>
</tr>
</tbody>
</table>

The optimal solutions for the deterministic case when \( c(e^*_1) \) is constrained and a two-part tariff contract is used are as follows:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Case OM</th>
<th>Case ODM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c(e^*<em>1) &lt; c</em>{min} )</td>
<td>( T_p &gt; \frac{(c_o - c_{min})(1-a_{1S}a_{2B})}{a_{1S}(p-c_o)} - a_{2S} )</td>
<td>( T_p &gt; \frac{(c_o - c_{min})(1-a_{1S}a_{2S})}{a_{1S}(p-c_o)} - a_{2S} )</td>
</tr>
<tr>
<td>( e^*_1 )</td>
<td>( \frac{c_0 - c_{min}}{\gamma Y c_0} )</td>
<td>( \frac{c_0 - c_{min}}{\gamma Y c_0} )</td>
</tr>
<tr>
<td>( e^*_2 )</td>
<td>( \frac{k p \delta_B}{2d_B} (p - c_{min}) )</td>
<td>( k p \delta_S (p - c_{min}) )</td>
</tr>
<tr>
<td>( w^*_n )</td>
<td>( c_{min} )</td>
<td>( p )</td>
</tr>
<tr>
<td>( F^*_n )</td>
<td>( (p - c_{min})(A - k p) + \frac{a_{2B}}{2} (p - c_{min})^2 \pi^B_i )</td>
<td>( -\pi^B_i )</td>
</tr>
<tr>
<td>( \pi^B_n )</td>
<td>( \pi^B_i )</td>
<td>( \pi^B_i )</td>
</tr>
<tr>
<td>( \pi^S_n )</td>
<td>( \frac{(c_0 - c_{min})^2}{2a_{1B}} - \frac{(c_0 - c_{min})^2}{2a_{1S}} )</td>
<td>( \frac{(c_0 - c_{min})^2}{2a_{1B}} - \frac{(c_0 - c_{min})^2}{2a_{1S}} )</td>
</tr>
</tbody>
</table>
### Theorem A2: Stochastic Demand Constrained Case

#### a) Wholesale pricing contract

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case OM</th>
<th>Case ODM</th>
</tr>
</thead>
<tbody>
<tr>
<td>If</td>
<td>$c(\hat{\epsilon}<em>{1,n}^*) &lt; c</em>{\text{min}}$</td>
<td>$a_{1S} &gt; \frac{c_{\text{min}} - c_{\text{om}}}{a_{2B}F(z_{\text{OM}}^<em>) + (A - kp + z_{\text{OM}}^</em>)}$</td>
<td>$a_{1S} &gt; \frac{c_{\text{min}} - c_{\text{om}}}{a_{2S}(p - c_{\text{om}}) + (A - kp + z_{\text{OM}}^*)}$</td>
</tr>
<tr>
<td>Hessian</td>
<td>$a_{1B} &lt; h_l$</td>
<td>$a_{2B} &lt; \frac{f'(x_{\text{OM}}^<em>)(A - kp + 2z_{\text{OM}}^</em>) + 2f'(z_{\text{OM}}^<em>)}{f'(z_{\text{OM}}^</em>)(p - c_{\text{min}}) - 2p(f'(z_{\text{OM}}^<em>))^2 - 2pF(z_{\text{OM}}^</em>)}$</td>
<td>$a_{2S} &lt; \frac{f'(z_{\text{ODM}}^<em>)(A - kp + 2z_{\text{ODM}}^</em>) + 2f'(z_{\text{ODM}}^<em>)}{f'(z_{\text{ODM}}^</em>)(p - c_{\text{om}}) - 2p(f'(z_{\text{OM}}^<em>))^2 - 2pF(z_{\text{OM}}^</em>)}$</td>
</tr>
<tr>
<td>$\hat{\epsilon}_{1,n}^*$</td>
<td>$c_0 - c_{\text{min}} \frac{c_0 - c_{\text{om}}}{c_0 - c_{\text{om}}}$</td>
<td>$c_0 - c_{\text{om}} \frac{c_0 - c_{\text{om}}}{c_0 - c_{\text{om}}}$</td>
<td>$c_0 - c_{\text{om}} \frac{c_0 - c_{\text{om}}}{c_0 - c_{\text{om}}}$</td>
</tr>
<tr>
<td>$\hat{\epsilon}_{2,n}^*$</td>
<td>$\frac{kp\delta_B}{2d_B}(p - c_{\text{om}})$</td>
<td>$a_{2B}F(z_{\text{OM}}^*) \frac{c_0 - c_{\text{min}}}{c_0 - c_{\text{om}}}$</td>
<td>$\frac{kp\delta_C}{2d_C}(pF(z_{\text{OM}}^*) - c_{\text{om}})$</td>
</tr>
<tr>
<td>$\hat{\omega}_{n}^*$</td>
<td>-</td>
<td>$pF(z_{\text{OM}}^*)$</td>
<td>$pF(z_{\text{OM}}^*)$</td>
</tr>
<tr>
<td>$\hat{z}_{n}^*$ solves</td>
<td>$\bar{F}(\hat{z}<em>{n}^*) = \frac{c</em>{\text{om}}}{p}$</td>
<td>$-pf(z_{\text{OM}}^<em>)(A - kp + z_{\text{OM}}^</em>) - 2p^2zf(z_{\text{OM}}^<em>)F(z_{\text{OM}}^</em>) + (p - c_{\text{om}})(1 + a_{2B}F(z_{\text{OM}}^<em>)) - pF(z_{\text{OM}}^</em>) = 0$</td>
<td>$\left(\frac{pF(z_{\text{OM}}^<em>)}{c_{\text{om}}}(1 - a_{2S}pf(z_{\text{OM}}^</em>)) - pf(z_{\text{OM}}^<em>)(A - kp + z_{\text{OM}}^</em>) = 0$</td>
</tr>
</tbody>
</table>

#### b) Two-part tariff contract

<table>
<thead>
<tr>
<th></th>
<th>Case OM</th>
<th>Case ODM</th>
</tr>
</thead>
<tbody>
<tr>
<td>If</td>
<td>$c(\hat{\epsilon}<em>{1,n}^*) &lt; c</em>{\text{min}}$</td>
<td>$a_{1S} &gt; \frac{c_{\text{min}} - c_{\text{om}}}{a_{2B}F(z_{\text{OM}}^<em>) + (A - kp + z_{\text{OM}}^</em>)}$</td>
</tr>
<tr>
<td>SOC</td>
<td>$-pf(z_{\text{OM}}^<em>)(1 + a_{2B}F(z_{\text{OM}}^</em>)) + (pF(z_{\text{OM}}^<em>) - c_{\text{om}})a_{2B}F'(z_{\text{OM}}^</em>) &lt; 0$</td>
<td>$-pf(z_{\text{OM}}^<em>)(A - kp + z_{\text{OM}}^</em>) + \frac{kp\delta_B}{2d_B}(p - c_{\text{om}})$</td>
</tr>
<tr>
<td>$\hat{\epsilon}_{1,n}^*$</td>
<td>$c_0 - c_{\text{min}} \frac{c_0 - c_{\text{om}}}{c_0 - c_{\text{om}}}$</td>
<td>$c_0 - c_{\text{om}} \frac{c_0 - c_{\text{om}}}{c_0 - c_{\text{om}}}$</td>
</tr>
<tr>
<td>$\hat{\epsilon}_{2,n}^*$</td>
<td>$\frac{a_{2B}F(z_{\text{OM}}^*)}{k\delta_B}$</td>
<td>$\frac{kp\delta_C}{2d_C}(p - c_{\text{om}})$</td>
</tr>
<tr>
<td>$\hat{\omega}_{n}^*$</td>
<td>$pF(z_{\text{OM}}^*)$</td>
<td>$pF(z_{\text{OM}}^*)$</td>
</tr>
<tr>
<td>$\hat{F}_{n}^*$</td>
<td>$-pF(z_{\text{OM}}^<em>)(A - k(1 - \delta_B\hat{\epsilon}_{2,n}^</em>)p + z_{\text{OM}}^<em>) + p\left(A - k(1 - \delta_B\hat{\epsilon}_{2,n}^</em>) - s(z_{\text{OM}}^*)\right) - d_B\hat{\epsilon}<em>{2,n}^2 - \hat{r}</em>{\text{p}}^2$</td>
<td>$-pF(z_{\text{ODM}}^<em>)(A - k(1 - \delta_B\hat{\epsilon}_{2,n}^</em>)p + z_{\text{OM}}^<em>) + p\left(A - k(1 - \delta_B\hat{\epsilon}_{2,n}^</em>) - s(z_{\text{ODM}}^*)\right) - d_B\hat{\epsilon}<em>{2,n}^2 - \hat{r}</em>{\text{p}}^2$</td>
</tr>
<tr>
<td>$\hat{z}_{n}^*$ solves</td>
<td>$(pF(z_{\text{OM}}^<em>) - c_{\text{om}})(1 + a_{2B}F(z_{\text{OM}}^</em>)) = 0$</td>
<td>$F(z_{\text{ODM}}^*) = \frac{c_{\text{om}}}{p}$</td>
</tr>
</tbody>
</table>
Appendix B

Proof of Theorem 1 and Theorem A1 (Case I):

This is a constrained optimization problem, so we use the Lagrangian and the KKT conditions.
\[ \mathcal{L}(e_1, e_2, \lambda) = (p - c(e_1))(A - k(1 - \delta_B e_2)p) - d_B(e_1^2 + e_2^2) + \lambda(c(e_1) - c_{\text{min}}) \]

There are two possible cases, \( \lambda = 0 \), and \( \lambda > 0 \) (so \( c(e_1) = c_{\text{min}} \)).

When \( \lambda = 0 \), \( c(e_1) \geq c_{\text{min}} \), and we can use the FOC to find the optimal solutions.
\[ \frac{\partial \mathcal{L}(e_1, e_2, \lambda)}{\partial e_1} = \gamma_B c_0(A - k(1 - \delta_B e_2)p + z) - 2d_B e_1 = 0 \]
which implies that:
\[ e_1 = \frac{\gamma_B c_0(A - k(1 - \delta_B e_2)p)}{2d_B}. \] (A1)

With respect to \( e_2 \), taking the first order condition we get:
\[ \frac{\partial \mathcal{L}(e_1, e_2, \lambda)}{\partial e_2} = -c_0 k p \delta_B (1 - \gamma_B e_1) + k p^2 \delta_B - 2d_B e_2 = 0 \]
which implies that:
\[ e_2 = \frac{k p \delta_B (p - c_o + \gamma_B c_0 e_1)}{2d_B}. \] (A2)

Using (A1) and (A2), the definition of \( a_{1B} \) and \( a_{2B} \) in Theorem 1, and simplifying, we get \( e_{1,l}^* \) and \( e_{2,l}^* \) in the theorem. Substituting these into (2) provides the optimal profits. When \( \lambda = 0 \), the constraint \( (c(e_1) - c_{\text{min}}) > 0 \) results in the condition \( T_p = \frac{A - k p}{p - c_o} \leq \frac{(c_o - c_{\text{min}})(1 - a_{1B} a_{2B})}{a_{1B}(p - c_o)} - a_{2B} \). The solutions for \( e_{1,l}^* \) and \( e_{2,l}^* \) are optimal if the Hessian of \( \pi(e_{1,l}^*, e_{2,l}^*) \) is negative semi definite where the Hessian is given by:
\[
\begin{pmatrix}
-2d_B & k p \delta_B \gamma_B c_0 \\
(k p \delta_B \gamma_B c_0) & -2d_B
\end{pmatrix}
\]

H1 = \(-2d_B < 0 \) and H2 > 0 leads to the condition \( 1 - a_{1B} a_{2B} > 0 \) which holds by assumption.

When \( \lambda > 0 \), \( T_p = \frac{A - k p}{p - c_o} \geq \frac{(c_o - c_{\text{min}})(1 - a_{1B} a_{2B})}{a_{1B}(p - c_o)} - a_{2B} \) and the solutions are given in Theorem A1 in Appendix A. This results in \( c(e_1) = c_{\text{min}} \), leading to \( e_{1,l}^* = \frac{c_o - c_{\text{min}}}{\gamma_B c_0} \). Using this with the first order condition for \( e_2 \) in (A2) results in \( e_{2,l}^* = \frac{k p \delta_B (p - c_{\text{min}})}{2d_B} \). The second order condition shows the function is concave in \( e_2 \).  

Proof of Theorem 2 and Theorem A1 (Cases OM and ODM):

First we show the wholesale pricing contract, and then the two-part tariff contract.

Case OM wholesale pricing contract:

The buyer’s problem is given in (7). The first order condition results in
\[ e_{2,OM}^*(w) = \frac{k p \delta_B (p - w)}{2d_B}. \] (A3)
The second order condition shows that the buyer’s problem is concave. Substituting this into the supplier’s problem in (8), we obtain a constrained optimization problem, and use the Lagrangian and the KKT conditions.

\[ \mathcal{L}(w, e_1, \lambda, \mu) = (w - c(e_1)) \left( A - k \left( 1 - \delta_B e_{2,OM}(w) \right) p \right) - d_S e_1^2 + \lambda (c(e_1) - c_{min}) + \mu (p - w) \]

Taking the first order condition of \( \mathcal{L}(w, e_1, \lambda, \mu) \) with respect to \( e_1 \), we obtain:

\[ \frac{\partial \mathcal{L}(w, e_1, \lambda, \mu)}{\partial e_1} = y_S c_0 \left( A - kp + a_{2B} (p - w) \right) - 2d_s e_1 - y_S c_0 \lambda. \] (A4)

Taking the first order condition of \( \mathcal{L}(w, e_1, \lambda, \mu) \) with respect to \( w \), we obtain:

\[ \frac{\partial \mathcal{L}(w, e_1, \lambda, \mu)}{\partial w} = \left( A - kp + a_{2B} (p - w) \right) - a_{2B} (w - c_0 + y_S c_0 e_1) - \mu. \] (A5)

There are four possible cases of the constraints depending on if they are binding or not (the first two cases for Theorem 2 and the other two for Theorem A1).

Case 1: When \( \lambda = 0, \mu = 0 \), we have the unconstrained solution. Setting \( \frac{\partial \mathcal{L}(w, e_1, 0, 0)}{\partial e_1} = 0 \) in (A4) and solving for \( e_1 \), then substituting this into \( \frac{\partial \mathcal{L}(w, e_1, 0, 0)}{\partial w} = 0 \) in (A5), we obtain

\[ w_{OM}^* = \left( \frac{1 - a_1 a_{2B}}{2 - a_1 a_{2B}} \right) \frac{(p + (A - kp)/a_{2B}) + c_0}{y_S c_0}. \] (A6)

Using (A3), (A4), and (A6) and solving for \( e_{1,OM}^* \) and \( e_{2,OM}^* \), we obtain the expressions in the theorem.

Case 2: When \( \mu > 0 \) and \( \lambda = 0 \), \( w_{OM}^* = p \). Using this and \( \frac{\partial \mathcal{L}(p, e_1, 0, 0)}{\partial e_1} = 0 \) in (A4), we obtain

\[ e_{1,OM}^* = \frac{y_S c_0 (A - kp)}{2d_S}. \] (A7)

Using (A7) and \( w_{OM}^* = p \), we substitute in to find \( e_{2,OM}^* = 0 \) and the optimal profits.

Case 3: When \( \mu = 0 \) and \( \lambda > 0 \), \( c(e_1) = c_{min} \), which means \( e_{1,OM}^* = \frac{c_0 - c_{min}}{y_S c_0} \). Using this and \( \frac{\partial \mathcal{L}(w, e_1, 0, 0)}{\partial w} = 0 \), we obtain \( w_{OM}^* = \frac{A - kp + a_{2B} (p + c_{min})}{2a_{2B}} \). Using this and \( e_{1,OM}^* \), we substitute in to find

\[ e_{2,OM}^* = \frac{a_{2B} (p - c_{min}) - (A - kp)}{2kp a_{2B}} \] and the optimal profits.

Case 4: When \( \mu > 0 \) and \( \lambda > 0 \), we have the case where both constraints hold and thus, \( w_{OM}^* = p \) and \( c(e_1) = c_{min} \), which means \( e_{1,OM}^* = \frac{c_0 - c_{min}}{y_S c_0} \). Using these, we substitute in to find \( e_{2,OM}^* = 0 \) and the optimal profits.

The profit expression is jointly concave in \( e_1 \) and \( w \) if the Hessian is negative semi-definite. The Hessian is

\[
\begin{pmatrix}
-2d_S & -a_{2B} y_S c_0 \\
-a_{2B} y_S c_0 & -2a_{2B}
\end{pmatrix}
\]

\( H_1 = -2d_S \) and \( H_2 > 0 \) iff \( 2 - a_1 a_{2B} > 0 \). This holds by assumption thus the KKT conditions provide the optimal solution.
Using the two conditions, we obtain the boundary conditions:

If \( c(e_1) \geq c_{\text{min}} \), then \( T_p \leq \left( \frac{(c_0-c_{\text{min}})(2-a_1sB)}{a_1s(p-c_o)} - a_2B \right) \). This holds in Theorem 2, while \( T_p > \left( \frac{(c_0-c_{\text{min}})(2-a_1sB)}{a_1s(p-c_o)} - a_2B \right) \) in Theorem A1. If \( w < p \), then \( \frac{1}{T_p} > \left( \frac{1-a_1sB}{a_2B} \right) \). This is the second case for OM in both Theorem 2 and Theorem A1.

**Case ODM wholesale pricing contract:**

The supplier maximizes her profits in (9). Similar to the OM case, this is a constrained optimization problem, however, in this case, the supplier’s profits are not concave in \( w \). Thus, \( w \) will be set to one of the endpoints, \( p \) or \( 0 \), where clearly choosing \( w^*_{ODM} = p \) will result in higher profits. The profit expression is jointly concave in \( e_1 \) and \( e_2 \), where the Hessian is the same as for case I, except with the parameters belonging to the supplier (S) instead of buyer (B).

The Lagrangian is:

\[
\mathcal{L}(w, e_1, \lambda) = \left( p - c(e_1) \right)(A - k(1 - \delta_B e_2)p) - d_se_1^2 + d_2e_2^2 + \lambda(c(e_1) - c_{\text{min}})
\]

There are again two cases, when \( c(e_1) \geq c_{\text{min}} \) and otherwise.

Case 1: The optimal solution for \( \lambda = 0 \), \( c(e_1) \geq c_{\text{min}} \), is shown in Theorem 2, based on the first order conditions of \( \frac{\partial \mathcal{L}(e_1,e_2,0)}{\partial e_1} = 0 \) and \( \frac{\partial \mathcal{L}(e_1,e_2,0)}{\partial e_2} = 0 \).

Case 2: For the constrained case, \( \lambda > 0 \), \( c(e_1) = c_{\text{min}} \), which means \( e^*_{1,ODM} = \frac{c_0-c_{\text{min}}}{\gamma sc_o} \). Using this and \( \frac{\partial \mathcal{L}(e^*_{1,ODM},e_2,\lambda)}{\partial e_2} = 0 \), we obtain \( e^*_{2,ODM} = \frac{kp\delta_s}{2ds}(p - c_{\text{min}}) \) as shown in Theorem A1.

Using the condition, we obtain the boundary condition:

If \( c(e_1) \geq c_{\text{min}} \), then \( T_p \leq \left( \frac{(c_0-c_{\text{min}})(1-a_1sZ_1)}{a_1s(p-c_o)} - a_2S \right) \). This holds in Theorem 1, while \( T_p \geq \left( \frac{(c_0-c_{\text{min}})(1-a_1sZ_1)}{a_1s(p-c_o)} - a_2S \right) \) in Theorem A1.

**Case OM Two-Part Tariff Contract:**

In this case, the buyer’s problem with respect to \( e_2 \) is identical to his problem under the wholesale pricing contract and thus \( e^*_{2,OM}(w) \) is given by (A3). Similarly the second order condition shows \( \pi^R_{OM}(e_2) \) is concave in \( e_2 \).

The supplier wants to induce the buyer to participate in the 2PT contract with the minimum fixed fee, thus

\[
\pi^R_{OM}(e_2) = (p - w)(A - k(1 - \delta_B e_2)p) - d_B e_2^2 - F = \pi^R_i
\]

or

\[
F = (p - c(e_1))(A - k(1 - \delta_B e_2)p) - d_B e_2^2 - \pi^R_i
\]

Substituting this into the supplier’s profit expression in (8) and using the Lagrangian, we obtain

\[
\mathcal{L}(w, e_1, \lambda) = \left( p - c(e_1) \right)(A - k(1 - \delta_B e^*_{2,OM}(w))p) - d_se_1^2 - d_B e^*_{2,OM}(w)^2 - \pi^R_i + \lambda(c(e_1) - c_{\text{min}})
\]
Case 1: When $e_1 \geq c_{min}$, and the first order conditions give us:

$$\frac{\partial L(w,e_1,\lambda)}{\partial e_1} = y_S c_0 (A - kp + a_{2B}(p - w)) - 2d_S e_1 - y_S c_0 \lambda$$  \hspace{1cm} (A8)

and

$$\frac{\partial L(w,e_1,\lambda)}{\partial w} = -(p - c_0 + y_S c_0 e_1) + (p - w)$$  \hspace{1cm} (A9)

Setting (A8) and (A9) equal to 0 and solving results in the expressions given in the Theorem. Note that $w_{OM}^* = c(e_{1,OM}^*)$. Using $e_{1,OM}^*$ and $w_{OM}^*$, we substitute in to find $e_{2,OM}^*$ and the optimal profits.

Case 2: When $e_1 = c_{min}$, and thus $e_{1,OM}^* = \frac{c_0 - c_{min}}{y_S c_0}$. The first order condition with respect to $w$ leads to $w_{OM}^* = c_{min}$. Substituting in, we obtain $e_{2,OM}^*$ and the optimal profits.

The Hessian is

$$\begin{pmatrix} -2d_S & -a_{2B} y_S c_0 \\ -a_{2B} y_S c_0 & -a_{2B} \end{pmatrix}$$

$H1 = -2d_S$ and $H2 > 0$ iff $1 - a_{1S} a_{2B} > 0$. This holds by Remark 1 thus the solution is optimal.

**Case ODM Two-Part Tariff Contract (2PT)**

The supplier wants to induce the buyer to participate in the 2PT contract with the minimum fixed fee, thus

$$\pi_{ODM}^B = (p - c(e_1))(A - k(1 - \delta_S e_2)p) - F = \pi_f^B$$

or

$$F = (p - c(e_1))(A - k(1 - \delta_S e_2)p) - \pi_f^B$$

Substituting this into the supplier’s profit expression (9), we obtain

$$L(w,e_1,e_2,\lambda) = (p - c(e_1))(A - k(1 - \delta_S e_2)p) - d_S (e_1^2 + e_2^2) - \pi_f^B$$

Case 1: When $e_1 \geq c_{min}$, and the first order conditions give us the same efforts as in the wholesale case. To determine the optimal $w$ and $F$, note that because the supplier does not want to give the buyer more than the centralized profits $\pi_f^B$, $w_{ODM}^* = p$. Substituting in, we find the optimal values as given in the Theorem.

Case 2: When $e_1 = c_{min}$, so $e_{1,OM}^* = \frac{c_0 - c_{min}}{y_S c_0}$. Again because the supplier does not want to give the buyer more than the centralized profits $\pi_f^B$, $w_{ODM}^* = p$. Substituting in, we find the optimal values as given in the Theorem.

The profit expression is jointly concave in $e_1$ and $e_2$, where the Hessian is the same as for case I, except with the parameters belonging to the supplier (S) instead of buyer (B). □

**Proof of Theorem 3:**

a) As discussed in the text prior to the Theorem, the condition, $1/T_p \leq (1 - a_{1S} a_{2B})/a_{2B}$ guarantees that we are in case OM where $w_{OM}^* < p$. For the buyer to outsource we need:
$$\pi^B_{OM} = \frac{(p-c^2)^2}{2(2-a_1a_B)^2}(a_{2B} + 2T_p - \frac{(3-a_1a_{2B})(1-a_1a_{2B})T_p^2}{a_{2B}}) > \frac{(p-c^2)^2}{2(1-a_1a_{2B})}(a_{2B} + 2T_p + a_{1B}T_p^2) = \pi^B_I.$$ Simplifying and rearranging the result follows.

If this does not hold, or if \( \frac{1}{T_p} \leq \frac{(1-a_1a_{2B})}{a_{2B}} \), then \( w^*_OM = p \) and the buyer would make 0 in profits, thus outsourcing will not occur.

b) In case ODM, where there is a wholesale pricing contract, \( w^*_ODM = p \), so the buyer would make 0 in profits, and outsourcing will never occur. □

**Proof of Theorem 4:**

With a two-part tariff contract, the buyer is indifferent to outsourcing because his profits under outsourcing are equal to those under the centralized case. However, the supplier will only offer a two-part tariff contract if her profits are greater than 0. Rearranging and simplifying \( \pi^S_{OM} > 0 \) and \( \pi^S_{ODM} > 0 \) leads to the conditions in parts a) and b) of the Theorem, respectively. □

**Proof of Theorem 5:**

Using the optimal efforts from Theorem 1 for case I and the optimal two-part tariff efforts from Theorem 2 for cases OM and ODM, we rearrange and simplify \( e^*_1, n > e^*_2, n \) for \( n = I, OM \) and ODM, obtaining the conditions in the theorem. □

**Proof of Lemma 1:**

For \( i = 1,2 \) and \( j = B, S \), we obtain the following.

\[
\frac{\partial G_{ij}}{\partial y_j} = \frac{8d_jy_j(c_0k_p\delta_j)^2}{(2d_j^2-(y_jc_0k_p\delta_j))} \geq 0
\]

\[
\frac{\partial G_{ij}}{\partial \delta_j} = \frac{8d_j\delta_j(y_jc_0k_p)^2}{(2d_j^2-(y_jc_0k_p\delta_j)^2)} \geq 0
\]

\[
\frac{\partial G_{ij}}{\partial d_j} = -32d_j^2 - 4(y_jc_0k_p\delta_j)^2 \leq 0. \quad \square
\]

**Proof of Theorem 6:**

For each part a) b) and c), we pairwise compare the unconstrained optimal efforts for the two-part tariff contracts, \( e^*_1, n, n = I, OM, ODM \), by rearranging, simplifying, and using the definition of the Gap function \( G_i, i = 1,2. \) □
Proof of Theorem 7 and Theorem A2:
(a) Case I:
Similar to Theorem 1, this is a constrained optimization problem and thus we use the Lagrangian and the KKT conditions.

\[ L_i(z, e_1, e_2) = -c(e_1)(A - k(1 - \delta_B e_2)p + z) + p(A - k(1 - \delta_B e_2)p - s(z)) - d_B(e_1^2 + e_2^2) + \lambda(c(e_1) - c_{\text{min}}) \]

There are two possible cases, \( \lambda = 0 \), and \( \lambda > 0 \).

Case 1: When \( \lambda = 0 \), \( c(e_1) \geq c_{\text{min}} \) we use the FOC to find the optimal solutions.

\[ \frac{\partial L_i(z, e_1, e_2)}{\partial e_1} = \gamma_B c_0(A - k(1 - \delta_B e_2)p + z) - 2d_B e_1 = 0 \]

which implies that:

\[ e_1 = \frac{\gamma_B c_0(A - k(1 - \delta_B e_2)p + z)}{2d_B}. \tag{A10} \]

With respect to \( e_2 \), taking the first order condition we get:

\[ \frac{\partial L_i(z, e_1, e_2)}{\partial e_2} = -c_0kp\delta_B(1 - \gamma_B e_1) + kp^2\delta_B - 2d_B e_2 = 0 \]

which implies that:

\[ e_2 = \frac{kp\delta_B(p - c_0 + \gamma_B c_0 e_1)}{2d_B}. \tag{A11} \]

Using (A10) and (A11), the definition of \( a_{1B} \) and \( a_{2B} \) in Theorem 1, and simplifying, we get \( \hat{e}_{1,i}^* \) and \( \hat{e}_{2,i}^* \) in the theorem.

Taking the first partial derivative with respect to \( z \) we get:

\[ \frac{\partial L_i(z, e_1, e_2)}{\partial z} = -c_0(1 - \gamma_B e_1) + p\bar{F}(z) = 0. \tag{A12} \]

Using \( \hat{e}_{1,i}^* \) and \( \hat{e}_{2,i}^* \) and rearranging (A12), we get \( \bar{z}_i^* \).

Note that because \( c(e_{1,i}) \) must be larger than \( c_{\text{min}} \), we simplify \( c(\hat{e}_{1,i}^*) > c_{\text{min}} \), giving us

\[ a_{1B} < \frac{c_{\text{min}}}{\gamma_B c_0 (p - c_{\text{min}})(\gamma_B c_0)^2} \]

Case 2: When \( \lambda > 0 \), \( c(e_1) = c_{\text{min}} \) and accordingly \( \hat{e}_{1,i}^* = \left( c_0 - c_{\text{min}} \right) / (\gamma_B c_0) \).

The first order condition with respect to \( z \) results in \( \bar{F}(\bar{z}_i^*) = c_{\text{min}} / p \). The first order condition for \( e_2 \) is

\[ \frac{\partial L_i(z, e_1, e_2)}{\partial e_2} = -c_0kp\delta_B(1 - \gamma_B e_1) + kp^2\delta_B - 2d_B e_2 = 0 \]

results in \( \hat{e}_{2,i}^* \) given in the Theorem.

The solutions for \( \hat{e}_{1,i}^* \), \( \hat{e}_{2,i}^* \), and \( \bar{z}_i^* \) are optimal if the Hessian of \( \bar{F}(\hat{e}_{1,i}^*, \hat{e}_{2,i}^*, \bar{z}_i^*) \) is negative semi definite where the Hessian is given by:

\[
\begin{pmatrix}
-2d_B & kp\delta_B\gamma_B c_0 & \gamma_B c_0 \\
kp\delta_B\gamma_B c_0 & -2d_B & 0 \\
\gamma_B c_0 & 0 & -pf(z)
\end{pmatrix}
\]
H1 = −2d_B, which is always negative. H2 results in the condition \(4d_B^2 > (k p \delta_B \gamma_B c_0)^2\), which simplifies to \(a_{1B} a_{2B} < 1\) which holds by remark 1. H3 results in the condition \(4d_B^2 > (k p \delta_B \gamma_B c_0)^2 + \frac{2d_B (y_B c_0)^2}{p f(z)}\) which simplifies to \(a_{1B} < h_i = \frac{pf(z)}{a_{2B}p f(z)}\), the condition from the theorem.

**Case OM (Wholesale pricing contract):**

For the buyer’s problem in (16), we take the first partial derivatives with respect to \(z\) and \(e_2\) and get:

\[
\frac{\partial \pi_B^B(z, e_2)}{\partial z} = -w + p f(z) = 0 \tag{A13}
\]
\[
\frac{\partial \pi_B^B(z, e_2)}{\partial e_2} = k p^2 \delta_B - k p \delta_B w - 2d_B e_2 = 0. \tag{A14}
\]

Rearranging (A13) and (A14), we obtain \(\hat{e}_{2,OM}^*\) and \(\hat{z}_{OM}^*\). The solutions for \(\hat{e}_{2,OM}^*\) and \(\hat{z}_{OM}^*\) are optimal if the Hessian \(\pi_{OM}^B(\hat{e}_{2,OM}^*, \hat{z}_{OM}^*)\) is negative semi definite, where the Hessian is given by:

\[
\begin{pmatrix}
-p f(z) & 0 \\
0 & -2d_B
\end{pmatrix}
\]

H1 = \(-p f(z)\), which is always negative. H2 = \(2pd_B f(z)\), which is always positive. Thus the Hessian is negative semi definite.

For the supplier’s problem in (17), we use the Lagrangian and constrained optimization where there are two possible cases, \(\lambda = 0\), and \(\lambda > 0\). We substitute in the buyer’s decisions and solve using \(z\) instead of \(w\), by using (A4).

**Case 1:** When \(\lambda = 0\), \(c(e_1) \geq c_{min}\) we use the FOC with respect to \(e_1\) and \(z\) to find the optimal solutions.

\[
\frac{\partial \pi_{OM}(w(z), e_1)}{\partial e_1} = \gamma_S c_0 (A - k (1 - \delta_B \hat{e}_{2,OM}^*)) p + z - 2d_S e_1 = 0. \tag{A15}
\]
\[
\frac{\partial \pi_{OM}(e_1, w(z))}{\partial z} = -p f(z) (A - k (1 - \delta_B \hat{e}_{2,OM}^*)) p + z + (p \tilde{F}(z) - c_0 (1 - \gamma_S e_1)) (a_{2B} p f(z) + 1) = 0. \tag{A16}
\]

Rearranging using \(\hat{e}_{2,OM}^*\), (A15), (A16) and the definition of \(a_{1S}\) in Theorem 1, we obtain \(\hat{e}_{1,OM}^*\) and \(\hat{z}_{OM}^*\). Note that because \(c(e_1)\) must be larger than \(c_{min}\), we simplify \(c(\hat{e}_{1,OM}^*) > c_{min}\) to obtain

\[
a_{1S} < \frac{c_0 - c_{min}}{a_{2B} p f(z_{OM}^*) + (A - k p + z_{OM}^*)},
\]

The solutions for \(\hat{e}_{1,OM}^*\) and \(z_{OM}^*\) are optimal if the Hessian of \(\pi_{OM}^S(\hat{e}_{1,OM}^*, \hat{z}_{OM}^*)\) is negative semi definite where the Hessian is given by:

\[
\begin{pmatrix}
-2d_S & \gamma_S c_0 (a_{2B} p f(z) + 1) \\
\gamma_S c_0 (a_{2B} p f(z) + 1) & -2pf(z) (a_{2B} p f(z) + 1) - a_{2B} p^2 f'(z) F(z) (2 - a_{1S} a_{2B}) - p f'(z) (1 - a_{1S} a_{2B}) (A - k p + z) + a_{2B} p f'(z) (p - c_0)
\end{pmatrix}
\]

H1 = \(-2d_S\), which is always negative. H2 results in the condition given in the theorem:

\[
a_{1S} < \pi_{OM}^S = \frac{2p f(z) (1 + a_{2B} p f(z)) + pf'(z) (2 a_{2B} p f(z) + (A - k p + z) - a_{2B} (p - c_0))}{(1 + a_{2B} p f(z) - a_{2B} p f'(z)) (A - k p + z) + a_{2B} p f'(z)}.
\]
Case 2: When \( \lambda > 0 \), \( c(e_1) = c_{\min} \) and accordingly \( \hat{e}_{1,OM}^* = \left( c_0 - c_{\min} \right) / \left( \gamma_S c_0 \right) \) and the buyer’s problem is unchanged.

The supplier’s problem becomes:

\[
\max_{z} \hat{h}_{OM}^S(z) = (p \tilde{F}(z) - c_{\min}) \left( A - k \left( 1 - \delta_B \hat{e}_{2,OM}^* \right) p + z \right) - d_S \hat{e}_{1,OM}^*^2
\]

s.t. \( \alpha \leq z \leq \beta \)

The first order condition for \( z \) is:

\[
\frac{\partial \hat{L}_{OM}^S(z)}{\partial z} = -pf(z)(A - k(1 - \delta_B \hat{e}_{2,OM}^*)p + z) + (p \tilde{F}(z) - c_{\min})(1 + a_{2B}pf(z)) = 0,
\]

which results in the expression for \( \hat{z}_{OM}^* \) in Theorem A2.

\( \hat{z}_{OM}^* \) is optimal if the second order condition is less than 0, which as shown in Theorem A2 results in:

\[
a_{2B} < \frac{f'(\hat{z}_{OM}^*)(A-kp+\hat{z}_{OM}^*)^2}{f'(\hat{z}_{OM}^*)\left(f'(\hat{z}_{OM}^*)^2z_0 - 2pf(\hat{z}_{OM}^*)\right)}.
\]

**Case ODM (Wholesale pricing contract):**

For the buyer’s problem in (18), we take the first derivative with respect to \( z \) and get:

\[
\frac{\partial \hat{h}_{ODM}^B(z)}{\partial z} = -w + p \tilde{F}(z) = 0
\]

(A17)

For the supplier’s problem in (19), we use the Lagrangian and constrained optimization where there are two possible cases, \( \lambda = 0 \), and \( \lambda > 0 \). We substitute in the buyer’s decisions and solve using \( z \) instead of \( w \), by using \( w = p \tilde{F}(z) \) from buyer’s first order condition.

Case 1: When \( \lambda = 0 \), \( c(e_1) \geq c_{\min} \), and we use the FOC with respect to \( e_1, e_2 \), and \( z \) to find the optimal solutions:

\[
\frac{\partial \hat{L}_{ODM}(e_1,e_2,z)}{\partial e_1} = \gamma_S c_0 (A - k(1 - \delta_S e_2)p + z) - 2d_S e_1 = 0. \tag{A18}
\]

\[
\frac{\partial \hat{L}_{ODM}(e_1,e_2,z)}{\partial e_2} = kp \delta_S (p \tilde{F}(z) - c_0(1 - \gamma_S e_1)) - 2d_S e_2 = 0. \tag{A19}
\]

\[
\frac{\partial \hat{L}_{ODM}(e_1,e_2,z)}{\partial z} = -pf(z)(A - k(1 - \delta_S e_2)p + z) + (p \tilde{F}(z) - c_0(1 - \gamma_S e_1)) = 0 \tag{A20}
\]

Solving (A18) and (A19) for \( e_1 \) and \( e_2 \) results in \( \hat{e}_{1,ODM}^* \) and \( \hat{e}_{2,ODM}^* \), respectively. Using those in (A20) and rearranging results in \( \hat{z}_{ODM}^* \) in Theorem 7.

Note that because \( c(e_1) \) must be larger than \( c_{\min} \), we simplify \( c\left(\hat{e}_{1,ODM}^*\right) > 0 \) to obtain:

\[
a_{1S} < \frac{c_0 - c_{\min}}{a_{2S}(p \tilde{F}(\hat{z}_{ODM}^*) - c_{\min}) + (A - kp + \hat{z}_{ODM}^*)}.
\]

The solution for \( \hat{e}_{1,ODM}^*, \hat{e}_{2,ODM}^* \), and \( \hat{z}_{ODM}^* \) in the theorem are optimal if the Hessian \( \hat{h}_{ODM}^S(\hat{z}_{ODM}^*,\hat{e}_{1,ODM}^*,\hat{e}_{2,ODM}^*) \) is negative semi definite where the Hessian is given by:
H1 = -2dS, which is always negative. H2 results in the condition on a1S a2S < 1 which is satisfied by remark 1, and H3 results in the condition given in the theorem:

\[
a_{1S} < \frac{pf'(\hat{\nu}_{ODM}^*)(a_{2S}(pF(\hat{\nu}_{ODM}^*)-c_0)+(A-kp+\hat{\nu}_{ODM}^*))-a_{2S}(pF(\hat{\nu}_{ODM}^*))^2}{(1+a_{2S}pf(\hat{\nu}_{ODM}^*))^2}
\]

Case 2: When \( \lambda > 0 \), \( c(e_1) = c_{min} \) and accordingly \( \hat{\nu}_{1,ODM}^* = (c_0 - c_{min})/(\gamma SC_0) \) and the buyer’s problem is unchanged.

The supplier’s problem becomes:

\[
\max_{\hat{\nu}_{2,ODM}^*, e_2} (pF(z) - c_{min})(A - k(1 - \delta e_2)p + z) - dS \left( \hat{\nu}_{1,ODM}^* + e_2^2 \right) \\
\text{s.t.} \\
e_2 \geq 0 \\
\alpha \leq z \leq \beta
\]

The first order condition for \( z \) is:

\[
\frac{\partial L_{ODM}(e_2,z)}{\partial z} = -pf(z)(A - k(1 - \delta e_2)p + z) + (p\bar{F}(z) - c_{min}) = 0
\]

(A21)

The first order condition for \( e_2 \) is:

\[
\frac{\partial L_{ODM}(e_2,z)}{\partial e_2} = kp\delta_S(p\bar{F}(z) - c_{min}) - 2dSe_2 = 0.
\]

(A22)

Solving (A22) for \( e_2 \) results in \( \hat{\nu}_{2,ODM}^* \) as given in the theorem. Substituting \( \hat{\nu}_{2,ODM}^* \) into (A21) and rearranging gives the expression for \( z_{ODM}^* \) given in Theorem A2.

The solutions for \( \hat{\nu}_{2,ODM}^* \) and \( z_{ODM}^* \) in the theorem are optimal if the Hessian \( \hat{\nu}_{ODM}^* \) is negative semi definite where the Hessian is given by:

\[
\left( \begin{array}{ccc} -2dS & kp\delta_SySc_0 & \gamma SC_0 \\ kp\delta_SySc_0 & -2dS & -kp\delta_Sf(z) \\ \gamma SC_0 & -kp\delta_Sf(z) & -pf'(z)(A - k(1 - \delta e_2)p + z) - 2pf(z) \end{array} \right)
\]

H1 = -pf(z)(A - k(1 - \delta e_2)p + z) - 2pf(z), which is always negative. H2 results in the condition on \( a_{2S} \) given in Theorem A2.

b) Two-Part Tariff Contract

Case OM:

Taking the first partial derivatives of the buyer’s problem (16) with respect to \( e_2 \) and \( z \) and simplifying, we obtain:

\[
\bar{F}(\hat{\nu}_{OM}^*) = \frac{w}{p}, \text{ and } \hat{\nu}_{OM}^* = \frac{a_{2B}F(\hat{\nu}_{OM}^*)}{k\delta_B}.
\]

(A23)

Using (A23) we obtain the supplier’s problem. \( F_{OM}^* \) is found by setting the highest transfer fee given the buyer’s reservation profit, i.e., \( \hat{\nu}_{OM}^* \geq R_B \), where \( R_B \geq \hat{\nu}_{OM}^* \).
Case 1: When $\lambda = 0$, $c(e_1) \geq c_{\min}$. Taking the first derivative of the Lagrangian for the supplier’s problem with respect to $e_1$ and $z$, we obtain:

$$\frac{\partial L_{OM}(e_1,w(z))}{\partial e_1} = \gamma_S c_0 \left(A - k \left(1 - \delta_B \hat{e}_{2,OM}^*\right)p + z\right) - 2d_S e_1 = 0. \quad (A24)$$

$$\frac{\partial L_{OM}(e_1,w(z))}{\partial z} = - pf(z) \left(A - k \left(1 - \delta_B \hat{e}_{2,OM}^*\right)p + z\right) + \left(p \tilde{F}(z) - c_0 \left(1 - \gamma_S e_1\right)\right) (a_{2B}p f(z) + 1) = 0. \quad (A25)$$

Rearranging using (A23), (A24), (A25), and the definition of $a_{1S}$, we obtain $\hat{e}_{1,OM}^*$ and the expression for $z_{OM}^*$.

Note that because $c(e_1)$ must be larger than $c_{\min}$, we simplify $c(\hat{e}_{1,OM}^*) > c_{\min}$ to obtain

$$a_{1S} < \frac{c_0 - c_{\min}}{a_{2B}p F(z_{OM}^*) + (A - kp + z_{OM}^*)}.$$  

The solutions for $\hat{e}_{1,OM}^*$ and $z_{OM}^*$ are optimal if the Hessian of $L_{OM}(\hat{e}_{1,OM}^*, z_{OM}^*)$ is negative semi definite where the Hessian is given by:

$$
\begin{pmatrix}
-2d_S & \gamma_S c_0 (a_{2B}p f(z) + 1) \\
\gamma_S c_0 (a_{2B}p f(z) + 1) & - pf(z) (a_{2B}p f(z) + 1) + a_{2B}p f'(z)(p \tilde{F}(z) - c_0) \\
& + a_{1S} (a_{2B}p f(z) + 1)^2 + a_{1S} a_{2B}p f'(z)(a_{2B}p F(z) + (A - kp + z))
\end{pmatrix}
$$

H1 = $-2d_S$, which is always negative. H2 results in the condition given in the theorem,

$$a_{1S} < \frac{pf(z_{OM}^*) \left(1 + a_{2B}p f(z_{OM}^*)\right) - a_{2B}p f'(z_{OM}^*) \left(p \tilde{F}(z_{OM}^*) - c_0\right)}{2 \left(1 + a_{2B}p f(z_{OM}^*)\right)^2 + a_{2B}p f'(z_{OM}^*)}.$$  

Case 2: When $\lambda > 0$, $c(e_1) = c_{\min}$ and accordingly $\hat{e}_{1,OM}^* = (c_0 - c_{\min}) / (\gamma_S c_0)$ and the buyer’s problem is unchanged. $F_{OM}^*$ is found in the same way and is also unchanged. The supplier’s problem becomes:

$$\max_z \hat{L}_{OM}(z) = -c_{\min} \left(1 - \delta_B \hat{e}_{2,OM}^*\right)p + z + p \left(A - k \left(1 - \delta_B \hat{e}_{2,OM}^*\right)p + s(z)\right)$$

$$- d_S \left(c_0 - c_{\min}\right)^2 - d_B \left(\hat{e}_{2,OM}^*\right)^2 - n_i^B$$

s.t. $a \leq z \leq b$

Taking the first derivative with respect to $z$, we obtain:

$$\frac{\partial L_{OM}(z)}{\partial z} = -c_{\min} (a_{2B}p f(z) + 1) + p (a_{2B}p f(z) + \tilde{F}(z)) - \frac{2d_B a_{2B} f(z) F(z)}{k \delta_B} = 0.$$  

Rearranging results in the expression for $z_{OM}^*$ in the theorem. The solution is optimal if the second order condition is negative, which results in the condition in Theorem A2.

Case ODM:

Taking the first derivative of the buyer’s problem (18) with respect to $z$ and simplifying, we obtain:

$$F(z_{OM}^*) = \frac{w}{p}.$$  

(A26)
Using (A26) we obtain the supplier’s problem. \( \hat{F}^{*}_{ODM} \) is found by setting the highest transfer fee given the buyer’s reservation profit, i.e., \( \hat{F}^{B}_{ODM} \geq R_B \), where \( R_B \geq \hat{\pi}^{B}_i \).

When \( \lambda = 0 \), \( c(e_1) \geq c_{\min} \).

Taking the first order conditions of the supplier’s Lagrangian we get:

\[
\frac{\partial L^{S}_{ODM}(e_1,e_2)}{\partial e_1} = y_S c_0 (A - k(1 - \delta_S e_2)p + z) - 2d_S e_1 = 0. \tag{A27}
\]

\[
\frac{\partial L^{S}_{ODM}(e_1,e_2)}{\partial e_2} = k p \delta_S (p - c_0 (1 - y_S e_1)) - 2d_S e_2 = 0. \tag{A28}
\]

\[
\frac{\partial L^{S}_{ODM}(e_1,e_2)}{\partial z} = p F(z) - c_0 (1 - y_S e_1) = 0 \tag{A29}
\]

Solving (A27) and (A28) for \( e_1 \) and \( e_2 \) results in \( \hat{e}^{*}_{1,ODM} \) and \( \hat{e}^{*}_{2,ODM} \). Using \( \hat{e}^{*}_{1,ODM} \) in (A29) and rearranging results in the expression for \( \hat{z}^{*}_{ODM} \).

Note that because \( c(e_1) \) must be larger than \( c_{\min} \), we simplify \( c(\hat{e}^{*}_{1,ODM}) > c_{\min} \) to obtain

\[
a_{1S} < \frac{c_0 - c_{\min}}{a_{2SP} + (A - kp + \hat{z}^{*}_{ODM})}.
\]

The solution for \( \hat{e}^{*}_{1,ODM}, \hat{e}^{*}_{2,ODM}, \) and \( \hat{z}^{*}_{ODM} \) in the theorem are optimal if the Hessian of \( \hat{\pi}^{S}_{ODM}(\hat{e}^{*}_{1,ODM}, \hat{e}^{*}_{2,ODM}, \hat{z}^{*}_{ODM}) \) is negative semi definite where the Hessian is given by:

\[
\begin{pmatrix}
-2d_S & kp \delta_S y_S c_0 & y_S c_0 \\
kp \delta_S y_S c_0 & -2d_S & 0 \\
y_S c_0 & 0 & -p f(z)
\end{pmatrix}
\]

H1 = \(-2d_S \), which is always negative. H2 results in the condition \( 4d^2_S > (kp \delta_S y_S c_0)^2 \), which simplifies to \( a_{1S} a_{2S} < 1 \), which is satisfied by remark 1, and H3 results in the condition given in the theorem:

\[
a_{1S} < \frac{p f(\hat{z}^{*}_{ODM})}{(1 + 2a_{2SP} f(\hat{z}^{*}_{ODM}))}.
\]

When \( \lambda > 0 \), \( c(e_1) = c_{\min} \) and accordingly, \( \hat{e}^{*}_{1,ODM} = (c_0 - c_{\min})/(y_S c_0) \), and the buyer’s problem is unchanged. \( \hat{F}^{*}_{ODM} \) is found in the same way and is also unchanged. The supplier’s problem becomes:

\[
\max_{e_2} \hat{\pi}^{S}_{ODM}(e_2, z) = -c_{\min} (A - k(1 - \delta_S e_2)p + z) + p (A - k(1 - \delta_S e_2)p - s(z))
\]

\[
- d_S \left( \frac{(c_0 - c_{\min})^2}{y_S c_0} + e_2^2 \right) - \pi^{B}_i
\]

s.t. \( e_2 \geq 0 \)

\( a \leq z \leq \beta \)

Taking the first derivative with respect to \( z \), we obtain:

\[
\bar{F}(\hat{z}^{*}_{ODM}) = \frac{c_{\min}}{p}
\]

Taking the first derivative with respect to \( e_2 \) and rearranging, we obtain \( \hat{e}^{*}_{2,ODM} \). The solutions for \( \hat{e}^{*}_{2,ODM} \) and \( \hat{z}^{*}_{ODM} \) in the theorem are optimal if the Hessian of \( \hat{\pi}^{S}_{ODM}(\hat{z}^{*}_{ODM}, \hat{e}^{*}_{2,ODM}) \) is negative semi definite where the Hessian is given by:
\[
\begin{pmatrix}
- pf(z) & 0 \\
0 & -2d_s
\end{pmatrix}
\]

\[H_1 = -pf(z),\] which is always negative. \[H_2 = 2pf(\hat{z}_{ODM}^*)d_s > 0,\] thus the solutions are optimal. \[\Box\]

**Proof of Theorem 8:**

(a) Comparing the stochastic process innovation effort in Theorem 7 with the deterministic one in Theorem 1 for case I, we have, \[\hat{e}_{1,I}^* \geq e_{1,I}^*\] translating to:

\[
\frac{y_Bc_0(p-c_0)}{2d_B(1-a_{1B}a_{2B})}(a_{2B} + T_{p,I}) \geq \frac{y_Bc_0(p-c_0)}{2d_B(1-a_{1B}a_{2B})}(a_{2B} + T_p) \tag{A30}
\]

(A30) holds if and only if \(T_{p,I} \geq T_p\). Using the definition of \(T_p\) and \(T_{p,I}\) in (3) and (20), we get that this will hold if and only if \(\hat{z}_I^* \geq 0\).

The proof for the product innovation effort in case I, and both efforts for case ODM (2PT) follow similarly.

(b)(i) Using Theorems 2 and 7, \[\hat{e}_{1,OM}^* \geq e_{1,OM}^*\] translates to:

\[
\frac{ysc_0(p-\hat{w}_{OM}^*)}{2d_s}(a_{2B} + T_{p,OM}) \geq \frac{ysc_0(p-c_0)}{2d_s(1-a_{1S}a_{2B})}(a_{2B} + T_p) \tag{A31}
\]

Using the definition of \(T_p\) and \(T_{p,OM}\) in (3) and (20), (A31) can be simplified to:

\[
\hat{z}_{OM}^* \geq \left(\frac{1}{1-a_{1S}a_{2B}} - \frac{(p-\hat{w}_{OM}^*)}{p-c_0}\right)T_p a_{1S} > 0. \tag{A32}
\]

where the second inequality in (A32) is based on the fact that \(\hat{w}_{OM}^* > c_0\), and \(a_{1S}a_{2B} < 1\).

(b)(ii) Using Theorems 2 and 7, \[\hat{e}_{2,OM}^* \geq e_{2,OM}^*\] translates to:

\[
\frac{kps_B(p-\hat{w}_{OM}^*)}{2d_B} \geq \frac{kps_B(p-c_0)}{2d_B(1-a_{1S}a_{2B})}(1 + a_{1S}T_p) \tag{A33}
\]

Using (A33) and the definition of \(\hat{w}_{OM}^*\) in Theorem 7 and simplifying, the result follows. \[\Box\]