Designing Sustainable Products under Co-Production Technology

Yen-Ting Lin
School of Business, University of San Diego, 5998 Alcalá Park, San Diego, CA 92110, linyt@sandiego.edu

Haoying Sun
Mays Business School, Texas A&M University, 320 Wehner Building, College Station, TX 77843, hsun@mays.tamu.edu

Shouqiang Wang
Department of Management, College of Business and Behavioral Science, Clemson University, Clemson, SC 29634, shouqiw@clemson.edu

A firm makes its products through a co-production technology that utilizes a natural material with an exogenous distribution of vertically-differentiated quality grades. Along with a traditional product with a well-established quality standard, the firm also designs a green product made of lower-quality material at an additional cost. The market consists of two demand segments: traditional consumers who value only a product’s quality, and green consumers who additionally value the product’s material savings. We find that higher material cost and/or lower green product’s cost induce the firm to offer the green product. Demand from one (traditional or green) of the two consumer segments can be fulfilled by both products. Most notably, the firm may strategically unfulfill some traditional consumers’ demand. Perhaps unexpectedly, expansion of the green market may adversely result in higher resource consumption and waste. Counter-intuitively, the traditional product’s quality, when set by the firm, may increase as the material becomes more costly.

Key words: co-production; product line design; technology management; sustainability;

1. Introduction
Accelerated depletion of scarce natural resources has attracted unprecedented public attention in recent years. Consumers are also increasingly “green” and willing to pay for the public-good value, such as resource conservation, embedded in products (Kotchen 2006). For example, McKinsey finds that more than 70% of consumers would pay an additional 5% for environmentally friendly, or simply green, products (Miremadi et al. 2012) and Mintel Market Research (2010) reports that 35% of surveyed Americans would be willing to pay more for green products. As a result, the growth and opportunities in green markets have been deemed as “the next big thing” for small business (Murphy 2003).

Recently, co-production technology that utilizes variation within raw material to manufacturer different products has emerged as an innovative way to introduce green products.1 Each raw mate-

1 In the operations literature co-production refers to simultaneous production of quality-differentiated outputs from a single production run (Tomlin and Wang 2008, Chen et al. 2013, Bansal and Transchel 2014). In our context, co-
rial harvested from the nature may contain within it natural variation of physical properties (e.g., texture, color, density) which determine the final products’ quality levels (e.g., functionality, performance, aesthetics, durability). Such embedded variation also regulates the final products’ quantities that each unit of raw material can produce. The traditional manufacturing process may be wasteful in that only raw material with quality exceeding certain well-established standard is used to make products, while the remaining material with inferior quality is simply discarded. Co-production technology turns those inferior material into green products, thereby reducing waste and enhancing conservation of resource.

Green co-products emerged very recently in industries that typically rely on natural resources. It is best exemplified by Taylor Guitars, a premier acoustic guitar manufacturer headquartered in San Diego, CA. Ebony wood has been the primary raw material for fretboard (a.k.a. fingerboard), which sits at the top of a guitar’s neck. The uniformly black color is ebony’s signature trademark, making it the conventional choice for many musical instruments. As a less-known fact, only one out of ten ebony trees is pure black, while most ebony trees are actually “streaked” with a natural, continuous variability of tan-colored swirls. The streak level of the wood is non-discrete and can only be discerned through eyeballing after being harvested and acquired from the mill.2

Traditionally, guitar makers use only pure black, “streak-less”, ebony, and simply discard all of the ebony with streak, even though they can deliver the same acoustic quality as the streak-less ones after treatments (e.g., additional fillings, polishing and waxing) (Arnseth 2013). After years of such extravagant and wasteful practice, ebony becomes extinct in many parts of the world. Cameroon in Africa is the only remaining legal source for high-quality ebony, and the country is imposing a quota system to cap the total export of ebony (Kirlin 2012).

In 2014, Taylor Guitars made an adventurous move and overhauled 60% of its product line by using otherwise would-be discarded streaked ebony. The company even redesigned and dedicated a popular model, the 800 series, to 100% use of streaked ebony. Taylor Guitars also endeavored to raise consumers’ awareness through its magazines and reseller training, “convincing guitar buyers that variations in wood color, often perceived as flaws, are actually signs of sustainably harvested ebony” (White 2012). The company’s sustainable effort has been well-received by its customers and Taylor Guitars were even caught up in orders.

Fishery industry offers another example of sustainable co-products, where bycatch—the incidental catch of non-target aquatic species—pervasively endangers the water ecosystems. According to production is driven by variation of properties (e.g., texture, color, density) within each unit of the input. However, each product may require a different manufacturing process depending on the input properties. In that sense, co-production in our context is generalized to allow non-simultaneous production of products.

---

2 Taylor Guitars is not allowed to only acquire pure-back ebony. Instead, it has to acquire the wood with a quality distribution. This is even more the case after Taylor Guitars purchased and vertically integrated Crelicam, the largest ebony mill in Cameroon (Orsdemir et al. 2016).
the U.S. National Bycatch Report (Karp et al. 2011), commercial fishing in the U.S. produces 1.2 billion pounds of bycatch, and nearly one fifth of fish caught in U.S. waters are discarded due to wrong size, poor quality, or low market value. The food industries have recently invented green co-products that aim to explore the economic value of the bycatch (Dunn 2015). For instance, Houston-based fishmonger PJ Stoops sells the bycatch caught by the local fishermen to Houston’s creative restaurants as ingredients for their daily specials (Leschin-Hoar 2012). As another case, Miya’s Sushi, a well-known restaurant in Connecticut, has gained a strong following by offering innovative sushi made with bycatch invasive species instead of overfished species that are commonly used in sushi.³ Marine biologists (Zhou et al. 2010, Garcia et al. 2012, Zhou et al. 2015) argue that these green co-products may help reshape people’s dining habits and lead to more balanced harvesting to combat the bycatch problem. Also in the flooring industry, while hardwood flooring is made entirely with solid wood, engineered wood flooring combines a top layer of solid wood with a base that can be made out of wood scrap (Cochran 2016, Flooring.net). Hence, engineered wood flooring is a more sustainable option due to use of material that is disqualified for hardwood flooring (Noriega 2010, Anater 2011, Cochran 2016).

The use of co-production technology for green products leads to interesting research questions: How do co-production and the presence of green consumers shape the firm’s product line strategy? When should a firm introduce green products? How do constraint on resource availability (such as supply quota) and cost of raw material affect that decision?⁴ How does the production cost for green products affect the firm’s consumption and utilization of raw material?

To answer our research questions, we consider a monopolistic firm who acquires raw material with exogenous quality distribution for product manufacturing. The firm uses the portion of purchased material whose quality exceeds an existing standard to manufacturer a traditional product. Co-production technology allows the firm to make a green product using material that are disqualified for making the traditional product. The two products share the material cost, but the green product incurs an additional unit production cost compared to the traditional product. The market consists of two consumer segments: traditional and green consumers. Green consumers differ from traditional ones by additionally valuing the resource conservation enabled by the green product. Consumers are free to choose between the products according to their preferences. The firm determines its material ordering quantity, product prices and quality for the green product. Our model leads to the following insights for our research questions.

Intuitively, a firm would fulfill all of demand from a segment of consumers when they are willing to purchase a product. Surprisingly, we find that, in contrast, a profit maximizing firm may fulfill only


⁴ We use “raw material” and “natural resource” interchangeably throughout the paper.
some, and abandon the rest, of a consumer segment even when consumers in that segment obtain homogeneous value from the product. We call this phenomenon strategic abandonment. Even more surprisingly, when strategic abandonment happens, the firm opts to abandon traditional consumers who intend to purchase the more expensive (traditional) product, rather than abandoning the green consumers who are inclined to purchase the less expensive (green) product. This pattern of strategic abandonment is caused by co-production and existence of green consumers, and it happens when the raw material is costly and both the green product’s production cost and the size of green consumers are low. In fact, strategic abandonment occurs in the circumstances where co-production technology enables the firm to profitably enter the market when it would otherwise not do so.

One would also expect that having more green customers and enhancing their willingness-to-pay (WTP) for resource conservation would both encourage purchase of the green product, which in turn lower the consumption of resource from the environment. Interestingly, we find that resource consumption actually increases in the size of green consumers when strategic abandonment occurs; otherwise resource consumption decreases in the size of green consumers. In other words, corporate incentives to promote expansion of the green market may backfire, leading to higher consumption of natural resource. In contrast, increasing consumers’ WTP for resource conservation always reduces resource consumption. Therefore, having more green customers and enhancing their WTP for resource conservation may not yield the same implication on resource consumption as one would expect.

Imposing supply limit (e.g., export quota) and raising material cost (e.g., tax) are common regulatory instruments to contain resource consumption. When the green production cost is high, one may expect that both of the instruments would incentivize introduction of green products. We find that, however, imposing supply limit can induces the firm to introduce green products in situations where the price instrument is ineffective in doing so. Therefore, regulators who intend to encourage introduction of green products should impose supply limit instead of raising cost of resource.

Finally, advances in technologies help lower green production cost and hence encourage introduction of green products. Nevertheless, when the cost of raw material is high we show that reduction in green production cost may unintendedly lower utilization and increase consumption of raw material. In contrast, such unintended consequence does not arise when material cost is low.

The paper is organized as follows. In the next section, we first survey the pertinent literature. Section 3 presents our base model, where the traditional product’s quality is exogenously pre-specified. In §4, we characterize the manufacturer’s optimal product line strategy and discuss its

\footnote{We numerically show in Appendix that strategic abandonment still occurs when consumers have heterogeneous product valuation.}
implications. In §5, we examine two extensions of our base model. We conclude the paper in §6. All the proofs and some technical results are relegated to the appendices and the online supplement.

2. Literature Review

Our paper belongs to the long-lasting research on product line design dating back to the seminal work by Mussa and Rosen (1978) and Moorthy (1984) and nowadays evolving to a broader field of new product development (NPD). Lancaster (1990) and Krishnan and Ulrich (2001) offer comprehensive reviews of the literature in this area. This line of research views the design of product lines as a discrimination tool to profitably screen heterogeneous consumers, who are privately informed about their willingness-to-pay for vertically differentiated product qualities. Our model enriches this framework by establishing the linkage among products in a line, which is advocated by Krishnan and Ramachandran (2008).

Of particular interest are the works by Krishnan and Gupta (2001) and Krishnan and Zhu (2006), who likewise examine other important scenarios of linking products during the development process. Keenly motivated by different industry examples, Krishnan and Gupta (2001) consider a product-family sharing a common component/subsystem, which they refer to as platform, while Krishnan and Zhu (2006) consider development-intensive products (DIPs), for which the fixed development cost dominates the variable production cost. Their common key insight is that introducing a single standardized product for all consumers or a niche product catering only to the high-end consumers can be optimal (see also e.g., Bhargava and Choudhary 2008). This is in contrast with our finding that it is optimal to introduce two co-products (with possible purchase spills) or to abandon part of the high-end (traditional) segment while only catering to the low-end (green) segment.

Remanufacturing technology is another instance featuring a chronological interdependence between products of different quality levels: the quantity of previously sold new, high-quality product determines the supply of used, low-quality products that can be remanufactured. Several scholars (e.g., Debo et al. 2005, Ferrer and Swaminathan 2006, Atasu et al. 2008) examine how to leverage remanufactured products to segment the market.

Featuring the simultaneous interdependence between products of different quality levels, the co-production technology has been extensively studied in the operation management literature. Earlier researchers in this area (e.g., Bitran and Dasu 1992, Bitran and Leong 1992, Bitran and Gilbert 1994, Gerchak et al. 1996, Rao et al. 2004) typically take the product line decisions (i.e., quality grades and prices) as exogenously fixed and focus on the operational decisions (e.g., inventory, production schedule). In a two-product setting with exogenous quality grades, Tomlin and Wang (2008) endogenize the pricing decisions by explicitly modeling the customers’ utility function. Motivated by prevalent practices in industrial markets (e.g., semiconductor manufacturing), those
literature typically assumes downward substitution—unmet demand for a low-quality product is fulfilled by a higher-quality product at the low-quality price. In a two-product model, Bansal and Transchel (2014) relax that assumption by including the spill-up option, which is not easily generalizable to more than two products. They examine the optimal amount of downward substitution and identify the optimal strategy of withholding low-quality inventory in order to generate upward substitution.

Along this strand of literature, our work is closest to Chen et al. (2013), who examine a general product line design problem by endogenizing not only the quality and price decisions of each product but also the number of products in the line. Our model contrasts with theirs in three major aspects. First, Chen et al. (2013) preclude their customers from spill-up purchases, which allows them to represent their problem in an echelon structure. In contrast, we assume that the consumers are fully flexible (c.f., Zhu et al. 2014) in purchasing their products, making their methodology no longer applicable. Second, we include an additional marginal production cost for the low-quality green product, whereas Chen et al. (2013) only account for material cost. Last but not least, the customers in Chen et al. (2013) are only homogeneous in their marginal willingness-to-pay, whereas the consumers in our model are heterogenous with qualitatively different utility specifications.

We base our model of green consumers on Kotchen’s (2006) theory (see also e.g., Bagnoli and Watts 2003) that regards a green product as the joint provision of private goods that generate consumption value and (impure) public goods that produce social and environmental value. This theory has provided the underpinnings for numerous subsequent studies (e.g., Besley and Ghatak 2007, Kotchen and Moore 2007, Pecorino 2013). A substantial body of empirical research (e.g., Cason and Gangadharan 2002, Jensen et al. 2004, Hicks 2012, and the reference therein) quantifies the consumer willingness-to-pay for such environmental and social public goods embedded in various products.

More specifically, the green consumer in our model derives, in addition to the quality-dependent value resulted from her private consumption of the product, a public-good value that is proportional to quality differential between the product and the high-quality traditional product. This relative notion of green products have been long advocated by both academics as well as practitioners. For instance, according to Ottman (1998), “green is relative, describing products with less impact on the environment than their alternatives” (see also Jensen et al. 2004, Durif et al. 2010, Ottman 2011). Peattie (1995) makes similar distinction: relative green products reduce the harm they cause to society or environment, whereas absolute green products contribute to the improvement of society or the environment.

Our green consumer’s utility specification is fundamentally different from those driven by other supply technologies. For example, in Atasu et al.’s (2008) study on remanufacturing strategy, green
consumers are referred to as those consumers who value the remanufactured products the same as the new products. In their model, a larger green segment may thus hurt the firm’s profit, which never occurs in our setting.

In spirit, the way we model the green consumers (especially in our base model where the traditional quality is exogenous) resembles the approach initiated by Chen (2001), who examines a firm’s incentive to develop green products. He abstracts the inherent trade-off between performance-based quality and environmental quality by assuming the sum of these two quality dimensions to be a constant. Focusing on similar tradeoffs, several subsequent studies investigate the green product design issues in various contexts ranging from product upgradability (Agrawal and Ulku 2013), servicizing (Agrawal and Bellos 2016), and sharing (Bellos et al. 2016) to extended producer responsibility legislations (Subramanian et al. 2009, Gui et al. 2016, Huang et al. 2016).

Finally, our paper contributes to the burgeoning literature on sustainable operations. In particular, Lee (2012) considers using by-product synergy to render the waste stream into a profitable product. By-product technology is a sequential manufacturing process where the waste generated from the production of the main product is used as the input for a product in a market that does not directly compete with the main product. In contrast, co-products are simultaneously manufactured by using different quality segments of the same material, and compete in the same market. Therefore, the quality decisions and the cannibalization effects, which drive most of the results in our co-production setting, are absent in the by-production scenario.

3. Model
A monopolistic manufacturer, which we call the firm, acquires a natural resource (e.g., wood, fish) as the raw material to make its products. The units are normalized such that one unit of the raw material can make exactly one unit of the product. The firm incurs a cost $c$ to acquire each unit of the raw material. We denote the total quantity of raw material acquired as $Q$. The raw material is vertically differentiated by a single physical attribute (e.g., the color purity of the wood, the size of fish), which we term as the material quality. It is measured by a scalar $q \in [0,1]$, with higher numerical value of $q$ representing higher material quality. The natural variation of material quality is characterized by a continuous probability distribution of $q$ on $[0,1]$, whose cumulative distribution function is denoted as $F(q)$. For analytical simplicity, we assume $F(q)$ to follow a uniform distribution, i.e., $F(q) = q$ for $q \in [0,1]$.

The material quality translates to the product quality, which determines the product’s consumption value. Following the convention in the co-production literature (e.g., Chen et al. 2013), we say a product is of quality $q$ if the product is made of raw material with quality in $[q,q']$ for some $q' > q$; namely, the product quality is determined by the minimum material quality that goes into
that product.\footnote{Labeling the product quality as the minimum material quality entering the product also complies with the truth-in-advertising regulations (e.g., 15 U.S. Code §45 and the Federal Trade Commission Act of 1914), which require that claims of product quality must be truthful, cannot be deceptive or misleading, and must be evidence-based.} The industry has established a conventionally accepted product quality level, which we refer to as the \textit{traditional quality} and denote as $q_t \in (0,1)$. For instance, only the ebony wood with color purity above certain level qualifies for making the guitar fretboards. The conventional production practice only uses raw materials with quality in $[q_t,1]$ to make the \textit{traditional products}; any material with quality level below $q_t$ is discarded. Hence, we model the traditional quality $q_t$ as an exogenous parameter and we will endogenize it in §5.1.

Nowadays, some consumers become “green” in the sense that they derive consumption utility not only from using the product as a private good but also from their contribution to resource conservation as a public good (Kotchen 2006, Besley and Ghatak 2007). In response, the firm can introduce a \textit{green product} made of material quality in $[q_g,q_t]$ for some $q_g$ lower than the traditional quality $q_t$. Additional production procedures may be needed to process raw material of quality below $q_t$. Without loss of generality, we normalize the traditional product’s unit production cost to zero, and denote $k \geq 0$ as the green product’s unit production cost.

For any given green product’s quality $q_g$ and material quantity $Q$, the quantities of the traditional and green products are respectively given by

$$Q_t := Q \left[1 - F(q_t)\right] = Q(1-q_t), \quad \text{and} \quad Q_g := Q \left[F(q_t) - F(q_g)\right] = Q(q_t - q_g).$$

(1)

As such, the manufacturing process represented here is a co-production technology: the quantities of the traditional and green products are simultaneously determined by material quantity $Q$ through their quality decision $q_t$ and $q_g$ as well as the exogenous distribution of material quality $F(\cdot)$. Slightly different from the conventional co-production models (e.g., Chen et al. 2013, and the reference therein), the co-production technology in our context includes an additional unit production cost $k$ for the low-quality (i.e., green) product.

For exogenously given traditional product’s quality $q_t$, the profit-maximizing firm needs to decide the material quantity $Q$, the green product’s quality $q_g$, as well as both traditional and green products’ prices, which we denote as $p_t$ and $p_g$, respectively.

On the demand side, without loss of generality, we normalize the total market size to one and denote $n \in [0,1]$ as the fraction of green consumers (thus, $1 - n$ is the fraction of traditional consumers). To highlight the interplay between the presence of environmentally conscious consumers and the co-production technology in an analytically tractable manner, we assume that the market is heterogenous only with these two segments — consumers within each segment are homogeneous.
Both segments of consumers can purchase either product. They both enjoy private-good utility $v_u q$ from using a product of quality $q$, where $v_u > 0$ represents the consumers’ marginal willingness-to-pay for the private goods’ quality. While a traditional consumer only derives utility from the consumption of private goods, a green consumer derives the public-good utility $v_e (q_t - q)$ from the product of quality $q \leq q_t$, which is additive to the private-good utility. Here, $v_e \in (0, v_u)$ represents the green consumers’ marginal willingness-to-pay for the public goods, and the quality differential $q_t - q_g$ measures the green product’s public-good value in terms of resource conservation: For each unit of raw material, a product of quality $q \leq q_t$ can salvage $F(q_t) - F(q) = q_t - q$ units of raw material that would otherwise be discarded if only the traditional product of quality $q_t$ were offered. (Readers are referred to Section 2 for the literature justifying this modelling choice.) The assumption $v_e < v_u$ suggests that the private-good consumption is still the primary contribution to the green consumer’s utility and the public-good consumption is secondary (Chen 2001, Kotchen 2006). For expositional simplicity, we normalize $v_u = 1$ and abbreviate $v_e$ as $v < 1$.

As such, before paying the price, both consumer segments enjoy higher consumption utility from the traditional product than from the green product, while the green consumer enjoys a higher consumption utility from the green product than the traditional consumer does. Table 1 below summarizes the utility functions for the two consumer segments purchasing the two products, where a consumer’s monetary payment for a product enters additively as a disutility. All consumers are utility maximizers with unit demand. The default utility of purchasing nothing is normalized to zero.

<table>
<thead>
<tr>
<th></th>
<th>Traditional product ($q_t, p_t$)</th>
<th>Green product ($q_g, p_g$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional consumers</td>
<td>$q_t - p_t$</td>
<td>$q_g - p_g$</td>
</tr>
<tr>
<td>Green consumers</td>
<td>$q_t - p_t$</td>
<td>$q_g + v(q_t - q_g) - p_g$</td>
</tr>
</tbody>
</table>

### 4. Analysis
This section contains the core analysis of our base model: we first formulate the firm’s problem in §4.1 followed by its solution in §4.2; we also discuss the environmental implications of the firm’s optimal decisions in §4.3.

#### 4.1. Firm’s problem formulation
Since a consumer will make the purchase only if the resulted utility is non-negative, the consumer utility specification in Table 1 immediately leads to the optimal price $p_t^* = q_t$ for the traditional product, allowing the firm to fully extract the consumer’s surplus from purchasing the traditional products. On the other hand, the optimal price for the green product has two candidates: $p_g = q_g$
or \( p_g = q_g + v(q_t - q_g) \). We refer to the former as the *regular price* and the latter as the *premium price* for the green product. At the regular price, the firm fully extracts the traditional consumer’s surplus but leaves positive surplus to the green consumer from purchasing the green product. At the premium price, the firm extracts the green consumer’s surplus from purchasing the green product while excluding the traditional consumers from purchasing the green product. In particular, under both pricing strategies, a traditional (green) consumer weakly prefers a traditional (green) product over the other product.  

By the nature of co-production technology, the ratio between the quantities supplied for the two products, \( Q_t/Q_g = (1 - q_t)/(q_t - q_g) \), may not be equal to the ratio between the sizes of the two demand segments, \( (1 - n)/n \). In such a case, the traditional (green) consumers may spill down (up) to purchase green (traditional) products. Therefore, the firm have four potential revenue sources. Let \( R_{ij}^i \) denote the firm’s revenue from selling product \( i \in \{t, g\} \) to consumer segment \( j \in \{T, G\} \), where superscripts \( T \) and \( G \) indicate the traditional and green consumer segments, respectively. We characterize these four revenue sources in Table 2 below, where \( x^+ := \max(x, 0) \) and \( 1 \{ \cdot \} \) is the indicator function taking value of 1 (0) if its argument is true (false).  

<table>
<thead>
<tr>
<th>Demand segment</th>
<th>Purchased product</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>Traditional</td>
<td>( R_t^T := q_t \min {1 - n, Q_t } )</td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>( R_g^T := p_g \min {(1 - n - Q_t)^+, (Q_g - n)^+ } 1 { p_g = q_g } )</td>
</tr>
<tr>
<td>Green</td>
<td>Traditional</td>
<td>( R_t^G := q_t \min {(n - Q_g)^+, (Q_t - (1 - n))^+ } )</td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>( R_g^G := p_g \min {n, Q_g} 1 { p_g = q_g + v(q_t - q_g) } )</td>
</tr>
</tbody>
</table>

As a concrete example, the firm generates revenue from traditional consumers who purchase the green products, i.e., \( R_g^T > 0 \), only when all the following three conditions hold: (1) the supply of the traditional products is inefficient to satisfy all traditional consumers, \( Q_t < 1 - n \), (2) excess supply of green products remains after fulfilling the green segment’s demand, \( Q_g > n \), and (3) the traditional consumers, who are not fulfilled by the traditional products, are willing to purchase the green products, i.e., \( q_g - p_g \geq 0 \), suggesting the green products to be priced at a regular price \( p_g = q_g \). Therefore, \( R_g^T := p_g \min \{(1 - n - Q_t)^+, (Q_g - n)^+ \} 1 \{ p_g = q_g \} \). All the other revenue sources can be derived in a similar fashion by noting that the traditional product’s optimal price is always \( p_t^* = q_t \).

---

\(^7\) In fact, the conventional incentive compatibility constraints in the product line design literature automatically hold: \( q_t - p_t^* \geq q_g - p_g \) for the traditional consumer and \( q_g + v(q_t - q_g) - p_g \geq q_t - p_t^* \) for green consumers when \( p_g \in \{q_g, q_g + v(q_t - q_g)\} \).
Accounting for the firm’s acquisition cost \( cQ \) for raw material and production cost \( kQ_g \) for green products, we can thus formulate the firm’s problem as:

\[
\Pi^* = \max_{q_g, q_t, Q} R_T^T + R_G^T + R_t^T + R_g^G - cQ - kQ_g \\
\text{subject to } 0 \leq q_g \leq q_t, p_g \in \{q_g, q_g + v(q_t - q_g)\}, \text{ and } Q \geq 0.
\]  

\[ (P) \]

4.2. Firm’s optimal decisions

Our solution strategy for the firm’s problem \((P)\) is to transform it to an optimization problem with only the quality decision \( q_g \). To that end, we first identify the optimal material quantity \( q_g \) as a function of \( q_g \). As characterized in the next lemma, three qualitatively different fulfillment strategies emerge, breaking the firm’s original problem into three subproblems with a single decision variable \( q_g \). By solving these subproblems and comparing the firm’s corresponding optimal profits, we can eventually identify the global optimal solution to \((P)\), culminating in Proposition 1 as our main result.

**Lemma 1.** The firm enters the market (i.e., has positive production) if and only if

\[
c \leq \bar{c}(k, q_t, v) = \max_{q_g \in [0, q_t]} (1 - q_t)q_t + (q_t - q_g)[q_g + v(q_t - q_g) - k],
\]

in which case we can restrict the search for the optimal solution to the firm’s problem \((P)\) among the following three regions:

1. \( q_g \geq \frac{n - n}{1 - n}, p_g = q_g + v(q_t - q_g) \) and \( Q = \frac{1}{1 - q_g} \), whereby both segments’ demands are fulfilled with some green consumers spilling up to traditional products.

2. \( q_g \leq \frac{n - n}{1 - n}, p_g = q_g + v(q_t - q_g) \) and \( Q = \frac{n}{q_t - q_g} \), whereby the green segment’s demand is fulfilled without any spill and the traditional segment’s demand may be partially fulfilled.

3. \( q_g \leq \frac{n - n}{1 - n}, p_g = q_g \) and \( Q = \frac{1}{1 - q_g} \), whereby both segments’ demands are fulfilled with some traditional consumers spilling down to green products.

The condition \((2)\) for the firm to enter production in Lemma 1 is quite intuitive: For each unit of raw material, the firm incurs a total cost of \( c + k(q_t - q_g) \), including the cost of acquiring the material and the cost of making green products. In return, the firm can generate a revenue of \( (1 - q_t)q_t \) by selling \( 1 - q_t \) traditional products at price \( q_t \) and, at most, a revenue of \( (q_t - q_g)[q_g + v(q_t - q_g)] \) by selling \( q_t - q_g \) green products at the premium price \( q_g + v(q_t - q_g) \). Thus, the firm enters production if and only if the total marginal revenue from each unit of raw material dominates the corresponding marginal cost, i.e., \( (1 - q_t)q_t + (q_t - q_g)[q_g + v(q_t - q_g)] \geq c + k(q_t - q_g) \).

When the firm enters production, Lemma 1 identifies three possible scenarios for the firm’s optimal decision. To see how they emerge, we first notice that the firm never acquires raw material that leads to excess or insufficient supply of both traditional and green products. That is, it will
never be optimal either to have $Q_t > 1 - n$ and $Q_g > n$, or to have $Q_t < 1 - n$ and $Q_g < n$ under condition (2). Subsequently, it leaves us with two possible situations:

When $Q_t \geq 1 - n$ and $Q_g \leq n$, which suggests $q_g \geq \frac{q_t - n}{1 - n}$, the firm can make excess traditional products to fulfill some green demands. Since the green consumer is indifferent between a traditional product and a green product sold at the premium price, this strategy allows the firm to charge a premium price for the green product and only requires the firm to acquire just enough raw material to exactly fulfill the entire market, i.e., $Q(1 - q_g) = 1$. When restricted to the region, the firm sells $\frac{1 - q_t}{1 - q_g}$ traditional products at price $q_t$ and $\frac{q_t - q_g}{1 - q_g}$ green products at price $q_g + v(q_t - q_g)$, reducing the firm’s problem $(P)$ to

$$
\Pi_1(c, k, q_t, v, n) := \max_{q_g \in \left[\frac{q_t - n}{1 - n}, q_t\right]} \frac{1 - q_t}{1 - q_g} q_t + \frac{q_t - q_g}{1 - q_g} (q_g + v(q_t - q_g)) - \frac{c}{1 - q_g} - k \frac{q_t - q_g}{1 - q_g}. 
$$

Alternatively, when $Q_t \leq 1 - n$ and $Q_g \geq n$, which suggests $q_g \leq \frac{q_t - n}{1 - n}$, there is potential shortage of traditional products. If the firm acquires enough raw material such that the supply of traditional products matches the demand of the traditional segment, i.e., $Q_t = 1 - n$, the firm can optimally increase the green product’s quality $q_g$ so that the supply of green products also matches the demand of the green segment, i.e., $Q_g = n$, allowing the firm to keep charging premium price for the green product. This leads to $q_g = \frac{q_t - n}{1 - n}$, in which case the first two regions in Lemma 1 coincide.

Withholding this strategy, the firm can either (i) abandon the unfulfilled traditional demand so that it acquires just enough raw material to meet green demands, i.e., $Q_g = n$, while selling the green products at its premium price, or (ii) satisfy the unfulfilled traditional demand with green products sold at the regular price, by acquiring raw material quantity to cover the entire market, i.e., $Q(1 - q_g) = 1$. The former strategy corresponds to the second region in Lemma 1 and reduces the firm’s problem $(P)$ to

$$
\Pi_2(c, k, q_t, v, n) := \max_{q_g \in \left[\frac{q_t - n}{1 - n}, 1 - q_t\right]} \frac{n}{q_t - q_g} (1 - q_t)q_t + n (q_g + v(q_t - q_g)) - k \frac{cn}{q_t - q_g}, 
$$

while the latter strategy corresponds to the lemma’s third region and reduces the firm’s problem $(P)$ to

$$
\Pi_3(c, k, q_t, v, n) := \max_{q_g \in \left[\frac{q_t - n}{1 - n}, 1 - q_t\right]} \frac{1 - q_t}{1 - q_g} q_t + \frac{q_t - q_g}{1 - q_g} q_g - \frac{c}{1 - q_g} - k \frac{q_t - q_g}{1 - q_g}. 
$$

The three subproblems, $(S_1)$, $(S_2)$ and $(S_3)$, are of a single decision variable $q_g$. The solution to the subproblem that yields the highest profit determines the optimal solution to the firm’s problem $(P)$. While these subproblems are relatively straightforward to solve, a key technical challenge is to identify conditions, under which a subproblem dominates the other two.
Proposition 1. There exist a threshold \( c^* = c^*(q_i, v, n) \) independent of \( k \) and four thresholds \( k_i = k_i(c, q_i, v, n) \) \((i = 1, \ldots, 4)\) such that the optimal solution \((q_g^*, p_g^*, Q^*)\) to the firm’s problem \((P)\) is given by Table 3 below. In particular, (i) \( \bar{c} = \bar{c}(k, q_i, v) \) in (2) is decreasing in \( k \); (ii) \( k_1, k_2 \) and \( k_3 \) are linearly increasing in \( c \); (iii) \( \bar{c}(k_2(c^*, q_i, v, n), q_i, v) = c^* \) if \( n \leq q_i \); (iv) \( k_4 \) is nonlinearly increasing first and then decreasing in \( c \geq c^* \); (v) \( k_1 > k_2 > k_3 \geq k_4 \) with \( k_3 = k_4 \) if and only if \( c = c^* \); and (vi) \( \Gamma_1 := \max \left\{ n, \frac{2(1-v)n}{1+(1-2v)n} \right\} \leq \Gamma_2 := 1 - \left( \frac{1-n}{\sqrt{1-(1-v)n}^2 + \sqrt{n}} \right) . \)

<table>
<thead>
<tr>
<th>(k,c)-Region</th>
<th>Dominant Subproblem</th>
<th>Conditions</th>
<th>( q_g^* )</th>
<th>( p_g^* )</th>
<th>( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_0 )</td>
<td>((S_1))</td>
<td>( c \leq \bar{c}, \ k \geq k_1 )</td>
<td>( q_i )</td>
<td>( q_i )</td>
<td>( \frac{1}{1-q_i} )</td>
</tr>
<tr>
<td>( \Omega_1 )</td>
<td>((S_1))</td>
<td>( c \leq \bar{c}, \ k_1 \geq k \geq k_2 )</td>
<td>( 1 - \sqrt{(1-q_i)^2 + \frac{c-k(1-q_i)}{1-v}} )</td>
<td>( q_g^* + v(q_i - q_g^*) )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
<tr>
<td>( \Omega_2 )</td>
<td>((S_2))</td>
<td>( q_i \geq \Gamma_1, \ c^* \leq c \leq \bar{c}, \ k \geq k_4 )</td>
<td>( q_i - \sqrt{\frac{c-k(1-q_i)}{1-v}} )</td>
<td>( q_g^* + v(q_i - q_g^*) )</td>
<td>( \frac{n}{q_i - q_g^*} )</td>
</tr>
<tr>
<td>( \Omega_{12} )</td>
<td>((S_1)=(S_2))</td>
<td>( q_i \geq \Gamma_1, \ c \leq c^*, \ k_1 \geq k \geq k_3 )</td>
<td>( (q_i - n)/(1-n) )</td>
<td>( q_g^* + v(q_i - q_g^*) )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
<tr>
<td>( \Omega_3 )</td>
<td>((S_3))</td>
<td>( q_i \geq \Gamma_2, \ c \leq c^<em>, \ k \leq k_3 ) ( q_i \geq \Gamma_2, \ c \geq c^</em>, \ k \leq k_4 )</td>
<td>( 1 - \sqrt{(1-q_i)^2 + c-k(1-q_i)} )</td>
<td>( q_g^* )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
</tbody>
</table>

Proposition 1 shows that the production cost structure of the firm’s manufacturing technology determines up to five qualitatively different optimal strategies in designing and pricing its green product, leading to different fulfillment outcomes. Intuitively, the unit material cost \( c \) is recouped by adopting the co-production technology and drives up material utilization by lowering the green product’s quality, while the marginal green production cost \( k \) reduces the supply of the green co-product and brings the green product’s quality closer to that of the traditional product. To provide a holistic view of the interplay between these two forces, Figure 1 maps the firm’s strategies and their defining threshold values onto the \((k,c)\)-space.

As can be seen from Figure 1, the firm introduces the green co-product only when the marginal green production cost is lower than the threshold \( k_1 \), which is in term increasing in the material cost \( c \). Otherwise, the firm only offers the traditional product (in region \( \Omega_0 \)). Namely, higher material cost and higher green production efficiency provide the firm with substitutable incentives to introduce the green co-product.

When the green co-product is introduced in region \( \Omega_1 \), the marginal green production cost \( k \) is still high (i.e., \( k \in [k_2, k_1] \)) relative to the material cost \( c \) (i.e., \( k_2 \) is increasing in \( c \)): the traditional

8 Their explicit expressions are provided in Appendix C.
product not only fulfills the entire traditional consumer segment but also part of the green demands, while the green product is offered just to fulfill the rest of the green consumer segment. Namely, some green consumers spill up to purchase the traditional products.

As the green production becomes more efficient (i.e., $k \in [k_3, k_2]$) as long as the material cost is not too high (i.e., $c \leq c^*$), the firm completely segregate the two market segments by fulfilling them with two different products (i.e., region $\Omega_{12}$) and hence no purchase spills occur.

Under the spill-up and no-spill strategies, the green product does not cannibalize the traditional product by penetrating the traditional consumer segment, enabling the firm to sell the green product at a premium price $q_g^* + v(q_t - q_g^*)$ and to appropriate all the consumer surplus from the green consumer segment.

Two interesting, and perhaps surprising, strategies emerge as the marginal green product cost $k$ keeps decreasing and the material cost $c$ keeps increasing. In this case, the firm can and needs to avail itself of the green product to increase the material utilization and recoup the material cost. Therefore, when the supply of traditional products is limited (i.e., $q_t \geq \Gamma_1$ or $\Gamma_2$), the firm can either (i) fulfill part of the traditional consumer segment with the green product, which in term needs to be sold at regular price $q_g^*$, or (ii) the firm can still keep segregating the two market segments and charging the premium price for the green product. The former strategy dominates the latter only when the green production technology is most efficient and the material cost is not too high (region $\Omega_3$). The latter strategy corresponds to higher material cost (i.e., $c \geq c^*$) in region...
Ω2, where the firm acquires only enough material to fulfill the green consumer segment, leaving some traditional consumers unsatisfied. The boundary \( k_4 \), which turns out to be non-monotonic in the material cost \( c \), marks the firm’s tradeoff between these two strategies.

As such, the boundaries between regions in the \((c,k)\)-space are qualitatively different in nature. Boundaries \( k = k_1, k_2 \) and \( c = c^* \) emerge as the optimal solutions to subproblems reach the boundaries of their feasible sets. In particular, region \( \Omega_{12} \) is where subproblem \((S_1)\) and \((S_2)\) turn out to have the same \( q_g^* = \frac{q_t-n}{1-n} \) as solution and hence coincide with each other. In contrast, boundaries \( k = k_3 \) and \( k = k_4 \) arise from comparing the optimal profits \( \Pi_2(c,k,q_t,v,n) \) and \( \Pi_3(c,k,q_t,v,n) \) respectively of the subproblems \((S_2)\) and \((S_3)\). As demonstrated in the next two lemmas, peculiar non-monotonicity behaviors of the firm’s optimal decisions \( q_g^* \) and \( Q^* \) may emerge as the firm’s strategy shifts between \( \Omega_2 \) to \( \Omega_3 \).

**Corollary 1.** Ceteris paribus, \( q_g^* \) is non-increasing in \( c \) except for an upward discontinuity when \( k_4(c,q_t,v,n) = k \) and \( c^* \leq c < \hat{c} \); but \( q_g^* \) is non-decreasing in \( k \) except for a downward discontinuity at \( k = k_4 \) for \( c > \hat{c} \), where \( \hat{c} := \min \left\{ q_t - vq_t^2, (1-q_t)q_t + 4(1-v)n_2(1-n_2)(1-q_t)^2 \right\} \).

Intuitively, higher material cost \( c \) and lower marginal green production cost \( k \) induce the firm to offer “greener” co-products at a lower quality. As illustrated by the solid lines in Figure 2, the green product’s quality may need to be adjusted abruptly when the firm switches its product line strategy from (partially) fulfilling the traditional demand segment with the green product at the regular price (i.e., region \( \Omega_3 \)) to giving away some traditional demand while maintaining the green product at the premium price (i.e., region \( \Omega_2 \)). As can be seen from Figure 2(a), when the material cost \( c \) increases, the green product’s quality \( q_g^* \) can in fact be adjusted upward from region \( \Omega_3 \) to \( \Omega_2 \), leading to a decrease in material utilization. This also suggests that the length of the optimal product line defined as the difference between the traditional and green products’ qualities \( q_t - q_g^* \) may no longer be monotonic in material cost \( c \), different from Chen et al.’s (2013) study of co-product design.

Also counter-intuitively, the green product’s quality \( q_g^* \) and hence its price \( p_g^* \) can be adjusted downward and then become independent as the additional green production cost \( k \) increases from region \( \Omega_3 \) to \( \Omega_2 \) in Figure 2(b). In region \( \Omega_2 \), the firm chooses the green product’s quality in order to match the supply of green products with the size of green consumer segment, thus annihilating the effects of \( k \).

**Corollary 2.** Ceteris paribus, \( Q^* \) is non-increasing in \( c \); but \( Q^* \) is non-decreasing in \( k \) except for a downward discontinuity at \( k = k_4 \) whenever \( k_4(c,q_t,v,n) \) is decreasing in \( c \).
Figure 2  The effect of $c$ and $k$ on the firm’s optimal decision $q_0^*$ and $Q^*$ ($v = 0.3$, $n = 0.3$, $q_t = 0.7$).

The dashed lines in Figure 2 illustrate Corollary 2. While it is quite intuitive for the firm to acquire less material when the material itself becomes more expensive or the co-production technology becomes more efficient, Corollary 2 identifies the strategic abandonment of part of the traditional consumer segment as another approach to reduce the material consumption, even when the co-production technology is as efficient. Even more intriguingly, such an approach is effective if and only if the material cost $c$ is high enough so that the threshold $k_4(c, q_t, v, n)$ is decreasing in $c$.

4.3. Environmental implications

In this subsection, we turn to examine how consumer “greenness” affects the firm’s environmental performance. Our model captures consumers’ “greenness” in two dimensions: the percentage of the green consumers in the market $n$, and their willingness-to-pay for resource conservation as the public good $v$. To measure the environmental performance, we examine three metrics: (1) the resource consumption$^9$ measured by $Q^*$; (2) the resource utilization, denoted as $U^* = 1 - q_0^*$; and (3) the resource waste, denoted as $W^* := Q^* q_0^*$.

Through direct inspection of the firm’s optimal decision in Table 3, we summarize the effects of the consumer “greenness” on the above three environmental performance measures and contrast them with the firm’s profit $Π^*$ in Table 4 below, whose proof is provided in Appendix C.

Table 4 suggests that the firm’s three environmental performance measures defined above generally do not deteriorate and can actually improve in some cases, as the green consumers become

$^9$While the firm uses only a portion of the acquired material, $Q^*$ units of the raw materials have to be harvested from the nature.
Table 4  Effects of consumer “greenness” on the firm’s environmental performance and profitability.

<table>
<thead>
<tr>
<th>(k, c)-Region</th>
<th>Monotonicity in n</th>
<th>Monotonicity in v</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q^<em>$ $U^</em>$ $W^<em>$ $\Pi^</em>$</td>
<td>$Q^<em>$ $U^</em>$ $W^<em>$ $\Pi^</em>$</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>$-$ $-$ $-$ $-$</td>
<td>$-$ $-$ $-$ $-$</td>
</tr>
<tr>
<td>$\Omega_1$</td>
<td>$-$ $-$ $-$ $-$</td>
<td>$\downarrow$ $\uparrow$ $\downarrow$ $\uparrow$</td>
</tr>
<tr>
<td>$\Omega_2$</td>
<td>$\uparrow$ $-$ $\uparrow$ $\uparrow$</td>
<td>$\downarrow$ $\uparrow$ $\downarrow$ $\uparrow$</td>
</tr>
<tr>
<td>$\Omega_{12}$</td>
<td>$\downarrow$ $\uparrow$ $\downarrow$ $\uparrow$</td>
<td>$-$ $-$ $-$ $\uparrow$</td>
</tr>
<tr>
<td>$\Omega_3$</td>
<td>$-$ $-$ $-$ $-$</td>
<td>$-$ $-$ $-$ $-$</td>
</tr>
</tbody>
</table>

Notation: $\uparrow$: increasing  $\downarrow$: decreasing  $-$: invariant

more willing to pay for resource conservation. However, having a larger percentage of green consumers in the market does not always yield the same effect as one might intuitively expect. In particular, in region $\Omega_2$, higher percentage of green consumers ($n$) in the market in fact leads to higher resource consumption ($Q^*$) as well as higher resource waste ($W^*$). This happens because in this region, the firm acquires just enough raw material so that the supply of green products exactly matches the size of green market, i.e., $Q^* (q_*^r - q_*^g) = n$, even though the corresponding supply of traditional products does not suffice to fulfill all traditional consumers. Thus, the firm’s resource consumption and waste are determined by green market share $n$ in region $\Omega_2$. In contrast, the firm acquires $Q^* = 1/(1 - q_*^g)$ units of raw material in all the other regions so that every consumer purchases a product.

This possible adversarial effect of more green consumers (i.e., higher $n$) is even more pronounced in light of the fact that more green consumers or greener consumers (i.e., higher $v$) can necessarily improve the firm’s profitability, as shown in the $\Pi^*$ column in Table 4. In other words, the corporate incentives to promote the expansion of green market (e.g., through educational programs) may sometime backfire, leading to undesirable environmental consequences.

As the green market size $n$ increases, the firm may switch its optimal strategy between different $(k, c)$-regions, leading to possible discontinuities in resource consumption, utilization and waste as illustrated in Figure 3. Notably, the resource consumption and utilization can simultaneously increase as the percentage of green consumer segment becomes larger. This phenomenon occurs only when resource consumption and utilization are compared across, but not within, regions. In the numerical example of Figure 3, when the percentage of green consumers in the market $n$ increases from 0.2 (region $\Omega_2$) to 0.8 (region $\Omega_1$), both resource consumption (from 1.18 to 2.01) and utilization (from 46.9% to 49.7%) increase. Indeed, the firm faces a higher demand as it switches

---

10 This phenomenon is also known as the “Jevons paradox” (Alcott 2005, Hertwich 2005).
its optimal strategy from giving up some traditional consumer segment (in region $\Omega_2$) to fulfilling the entire market (in region $\Omega_1$). Moreover, the resource waste also goes up (from 0.62 to 1.01), because the increase of consumption outpaces that of utilization.

5. Extensions

In this section, we extend our base model by relaxing two assumptions: exogenous traditional product’s quality $q_t$ and unlimited raw material quantity $Q$. First, we endogenize $q_t$ as the firm’s decision. This extension corresponds to the situation where the firm has the leadership power to set the conventional quality standard in the market. Next, we consider the situation where the exploitation of scarce resources is regulated. For instance, Cameroon government imposes an annual export quota on the ebony wood.

5.1. Endogenous traditional product’s quality

In this subsection, the firm can choose both products’ qualities, $q_t$ and $q_g$, and their prices, $p_t$ and $p_g$, as well as the raw material quantity $Q$. In this case, the firm’s optimal decisions are obtained by additionally optimizing $q_t$ over the results given in Proposition 1. We summarize the complete characterization in Proposition A.1 of Appendix A.

We find that all of our key results in the base model continue to hold qualitatively. In particular, all the regions emerging in Table 3 still exist (with different boundaries). Having more green consumers (i.e., larger $n$) still results in higher resource consumption in region $\Omega_2$. The effects of the unit material cost $c$ and additional green production cost $k$ on the firm’s product line strategy also remain. With traditional product’s quality endogenized, we are interested in examining the impact of $c$ and $k$ on the optimal $q_t^*$. 
Proposition 2. When $q_t$ is endogenously chosen, the effects of $c$ and $k$ on the optimal product qualities are summarized in Table 5. In region $\Omega_1$, $q_t^*$ increases (decreases) in $c$ if $0.5 < v < 0.75$ ($v < 0.5$).

Table 5 Effects of costs $(c,k)$ on product qualities.

<table>
<thead>
<tr>
<th>$(k,c)$-Region</th>
<th>Monotonicity in $c$</th>
<th>Monotonicity in $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_0$</td>
<td>↓ ↓</td>
<td>– –</td>
</tr>
<tr>
<td>$\Omega_1$</td>
<td>↓ or ↑ ↑</td>
<td>↓ ↑</td>
</tr>
<tr>
<td>$\Omega_2$</td>
<td>↑ ↓</td>
<td>– –</td>
</tr>
<tr>
<td>$\Omega_{12}$</td>
<td>↓ ↓</td>
<td>– –</td>
</tr>
<tr>
<td>$\Omega_3$</td>
<td>↓ ↓</td>
<td>↓ ↑</td>
</tr>
</tbody>
</table>

Notation: ↑: increasing ↓: decreasing −: invariant

Table 5 demonstrates the effect of the cost structure on the firm’s optimal product qualities in each strategy region, whose global non-monotonicity is illustrated in Figure 4. Most notably, as the material cost $c$ increases from region $\Omega_3$ to region $\Omega_2$, the traditional product’s optimal quality $q_t^*$ changes from piecewise decreasing in $c$ to piecewise increasing.

![Figure 4](image-url)

(a) The effect of $k$ ($c = 0.2$). (b) The effect of $c$ ($k = 0.2$).

Figure 4 The firm’s optimal product qualities $q_t^*$ and $q_g^*$ ($v = 0.4$ and $n = 0.1$).

This change of the monotonicity property can be understood as follows. As the raw material cost increases, the firm acquires less raw material. Consequently, if the firm were to increase the traditional product quality $q_t^*$, the supply of the traditional product would become even lower,
either forcing some traditional consumers to purchase the green product or to leave without any purchase at all. When the material cost \( c \) becomes too high to retain all of the traditional consumers (region \( \Omega_2 \)), the firm would increase \( q^*_t \) to maximize the revenue appropriated from the remaining traditional consumers. On the other hand, when \( c \) is relatively low (region \( \Omega_3 \)), the firm is still profitable to fulfill the demand from both segments.

While the traditional product’s quality \( q^*_t \) may be increasing or decreasing in \( c \), the green product’s quality \( q^*_g \) always decreases in \( c \) in all of the regions, because this enables the firm to better utilize more expensive raw material. Moreover, the length of the firm’s optimal product line, \( q^*_t - q^*_g \), is always increasing in \( c \), as illustrated in Figure 4(b). As first noted by Chen et al. (2013), this phenomenon counters the result in a unit-production setting. However, in their study, the highest product quality nonetheless remains a constant. Namely, their result is equivalent to the descending monotonicity of the lowest product quality in \( c \). We establish this result with the highest product quality \( q^*_t \) decreasing in \( c \) even in the presence of additional green production cost \( k \). In addition, Figure 4(a) demonstrates the intuitive yet opposite effect of \( k \) on the length of the optimal product line: the length of the product line shrinks as the green production becomes less efficient.

5.2. Limited raw material availability

In this section, we extend our base model by imposing a cap \( \bar{Q} \) on the raw material quantity \( Q \). This essentially introduces a capacity constraint \( Q \leq \bar{Q} \) to the firm’s problem \((P)\). Of course, our previous results remain unaltered when the optimal material quantity \( Q^* \) obtained in §4.2 is below \( \bar{Q} \). Therefore, we restrict our attention in this section to the more meaningful case, in which the capacity constraint \( Q \leq \bar{Q} \) must be binding. Specifically, the firm acquires the material quantity \( Q^* = \bar{Q} \) when \( \bar{Q} \leq \min \{ Q^*_{S_1}, Q^*_{S_2}, Q^*_{S_3} \} \), where \( Q^*_{S_i} \) is the optimal material quantity in subproblem \((S_i)\) defined in §4.2.

Taking a similar solution procedure as in our base model, we solve the firm’s problem in this case by identifying and comparing three subproblems that only involve a single decision variable \( q_g \), the green product’s quality. With slight abuse of notation,\(^{11}\) we still denote them as \((S_i)\) for \( i = 1, 2, 3 \). Subproblem 1 corresponds to the situation where some green consumers spill up to traditional products; Subproblem 2 corresponds to the situation where none of the consumer segments spill; Subproblem 3 corresponds to the situation where some traditional consumers spill down to green products. The following proposition provides the firm’s optimal decisions for \( v \leq \frac{1}{2} \). When \( v > \frac{1}{2} \), we show in proof of Proposition 3 that the firm’s optimal decisions become trivial in some cases without adding qualitatively new insights.

\(^{11}\) These three subproblems have different feasible sets than those in §4.2, whose explicit formulations are given in Lemma E.1 of the online supplement.
Proposition 3. When \( v \leq \frac{1}{2} \) and the firm orders \( Q = \bar{Q} \), it prices the traditional product at \( p^*_t = q_t \). The green product’s optimal quality \( q^*_g \) and price \( p^*_g \) are given in Tables A.2 and A.3 of Appendix A.

The complete algebraic characterization of the green product’s optimal quality and price is quite involving. For illustrative purpose, we use Table 6 to demonstrate one representative case where \( q_t \geq \frac{2n}{1+n} \) and \( \frac{2n}{q_t} \leq \bar{Q} \leq \frac{1-n(1+\sqrt{v})}{1-q_t} \).

<table>
<thead>
<tr>
<th>Dominant Subproblem</th>
<th>Region</th>
<th>Conditions</th>
<th>( q_g^* )</th>
<th>( p_g^* )</th>
<th>Unfulfilled consumer segment?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Omega_3 )</td>
<td>( k \leq \tilde{k}_4 )</td>
<td>( 1 - 1/\bar{Q} )</td>
<td>( q^*_g )</td>
<td>Traditional No</td>
</tr>
<tr>
<td></td>
<td>( \Omega_2 )</td>
<td>( \tilde{k}_4 \leq k \leq \tilde{k}_2 )</td>
<td>( \frac{k+q_t}{2} )</td>
<td>( q_t - n/\bar{Q} )</td>
<td>Green No</td>
</tr>
<tr>
<td></td>
<td>( \Omega_0 )</td>
<td>( \tilde{k}_2 \leq k \leq \tilde{k}_1 )</td>
<td>( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q^<em>_g + v(q_t - q^</em>_g) )</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Having limited raw material supply introduces two new effects to our base model. First, the material cost \( c \) becomes a fixed sunk cost and no longer plays any role in the firm’s optimal product line decision, and hence we just need to focus on the effect of additional green production cost \( k \). Second, it is possible for the firm to leave both consumer segments with unfulfilled demand due to insufficient material supply. Otherwise, all the qualitative insights garnered through our base model still hold. For example, as \( k \) increases, the firm shifts its strategy from co-products (region \( \Omega_3 \) and \( \Omega_2 \)) to a single product (region \( \Omega_0 \)). In region \( \Omega_3 \), the green product is sold at a regular price so that the traditional consumers can spill down, whereas in region \( \Omega_2 \), the green product exclusively supplies to the green segment at a premium price.

6. Conclusion
In this paper, we consider a firm who uses a scarce natural resource as the raw material to make its products. On the supply side, the firm utilizes co-production technology to produce both traditional and green products, while incurring an additional production cost for the green product. On the demand side, consumers are heterogeneous in whether they value the resource conservation enabled by the green product: green consumers value resource conservation while traditional consumers do not. The firm makes product line decisions, including the products’ prices and qualities, and the raw material quantity.
Through the lens of technology management, we identify resource scarcity, measured by the raw material’s cost, and green production efficiency, measured by the green product’s additional production cost, as two drivers of the firm’s product line design decisions. We find that the firm may use the green product’s price and the raw material quantity to induce distinct demand fulfillment patterns: green consumers spill up to purchase traditional products, traditional consumers spill down to purchase green products, or no spill takes place.

Interestingly, strategic abandonment of some traditional consumers may arise when the raw material is very scarce and the green production is sufficiently efficient. In that case, expansion of the green consumer segment improves the firm’s profit but it nevertheless elevates consumption and waste of natural resource. In other words, the firm’s incentive to expand the green market may sometimes backfire, causing negative environmental impacts.

Our analytic results shed some light on Taylor Guitars, the company that initially motivates this research. The company’s green production efficiency is relatively high because streaked ebony requires only minor cosmetic treatments, and strategic abandonment of demand for the traditional product does not seem to occur, according to our conversation with Charlie Redden, the company’s director of supply chain. In this case, our analysis may offer a justification for the company’s endeavor to expand its green consumer base to improve its profitability while lowering ebony consumption. However, as the ebony price keeps rising, the demand for the traditional product may shrink. In that case, our results call for careful scrutiny on the company’s environmental impacts because larger green demand may adversely increase the ebony consumption.

The implications that our findings can potentially deliver to the policy makers abound. First, policies (e.g., subsidies, knowledge transfer, best practice dissemination) that foster process improvement for the green production are win-win for the private industry as well as for the environment. Second, environmental regulators have to be cautious in using quotas to cap resource usage, because it may induce firms not to fulfill the demand and in term reduce social welfare while the resource is not fully utilized. Third, policies (e.g., tax reduction) that reward firms’ environmental initiatives may need to be carefully reviewed as those initiatives might simply be propelled by the firms’ profitability objectives rather than the claimed environmental benefits.

Every research has its boundary, and ours is no exception. Our model applies to situations where manufacturers dominate the market. In a more competitive market, however, a distinct model needs to be developed. While beyond the scope of this paper, it could be an interesting direction for future exploration.

References


Appendix A: Characterization of the Firm’s Optimal Decisions in §5

Proposition A.1 (Firm’s optimal decisions for endogenous \( q_t \)). When the firm endogenizes its choice of \( q_t \), there exist threshold values \( c_1, c_2, c_3 \) and \( c_4 \) such that the firm earns positive profit if \( c \leq c_4 \). In that case, the firm sets the price \( p_t^* = q_t^* \) for the traditional product.

1. For \( v < \frac{3}{4} \) and \( k < \frac{1}{2} \), the firm’s optimal decisions are summarized in Table A.1.
2. For \( v \geq \min\{\frac{3}{4}, k + \frac{1}{2}\} \), region \( \Omega_1 \) does not exist. In region \( \Omega_2, q_t^* = 1 \) and \( q_g^* = 0 \). The firm’s optimal decisions in regions \( \Omega_0, \Omega_{12} \) and \( \Omega_3 \) are given in Table A.1.
3. For \( k \geq \frac{1}{2} \) and \( v < k + \frac{1}{2} \), only region \( \Omega_0 \) exists and the firm’s optimal decisions are given in Table A.1.

Table A.1 The firm’s optimal decisions when \( q_t \) is endogenized.

<table>
<thead>
<tr>
<th>Region in ((k,c))-space</th>
<th>Conditions</th>
<th>( q_t^* )</th>
<th>( q_g^* )</th>
<th>( p_g^* )</th>
<th>( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_0 )</td>
<td>( c \leq \min{k^2, c_4} )</td>
<td>1 - ( \sqrt{c} )</td>
<td>1 - ( \sqrt{c} )</td>
<td>( q_t )</td>
<td>( \frac{1}{1-q_t} )</td>
</tr>
<tr>
<td>( \Omega_1 )</td>
<td>( k^2 \leq c \leq \min{c_1, c_4}, v &lt; \frac{3}{4} )</td>
<td>1 + ( \frac{2n-1}{\sqrt{1-v}} \Delta_1 - \frac{k}{2(1-v)} )</td>
<td>1 - ( \Delta_1 )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
<tr>
<td>( \Omega_2 )</td>
<td>( c_2 \leq c \leq c_4, k \leq \frac{1-2n(1-v)}{2-n} )</td>
<td>( \frac{1}{2}(1+\Delta_2) )</td>
<td>1 - ( \Delta_2 )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
<tr>
<td>( \Omega_{12} )</td>
<td>( c_1 \leq c \leq \min{c_5, c_6}, k \leq \frac{1-2n(1-v)}{2-n} )</td>
<td>1 - ( (1-n)\frac{\sqrt{c}}{\Delta_3} )</td>
<td>1 - ( \sqrt{\frac{c}{\Delta_3}} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
<tr>
<td>( \Omega_3 )</td>
<td>( c_5 \leq c \leq c_7 )</td>
<td>1 - ( \frac{k}{2} - \Delta_4 )</td>
<td>1 - 2( \Delta_4 )</td>
<td>( q_g^* )</td>
<td>( \frac{1}{1-q_g^*} )</td>
</tr>
</tbody>
</table>

\[ \Delta_1 = \sqrt{\frac{4c(1-v)-k^2}{3-4v}}, \Delta_2 = \sqrt{\frac{4c-1}{3-4v}}, \Delta_3 = 1 - n + n^2(1-v), \Delta_4 = \sqrt{\frac{4c-k^2}{12}} \]

\[ c_1 = \frac{k^2\Delta_3}{(2-n)^2}, c_2 = \min\{c_6, c_7\}, c_3 = \min\{c_5, c_6\}, c_4 = \begin{cases} \frac{1-\frac{k^2\Delta_3}{(2-n)^2}}{3-4v} & \text{if } k \leq \frac{1}{2} \text{ and } v < \frac{3}{4} \\ v - k & \text{if } v \geq \max\{\frac{3}{4}, k(k+1)\} \\ \frac{1}{2} & \text{otherwise} \end{cases} \]

\[ c_5 = \frac{k^2(1-5n^2+2n+3(2-2v)+4n^2(1-v)+2(1-2n)n\sqrt{\Delta_3}v)}{(1-4n^2+4(1-v)^2)} \]

\[ c_6 = \begin{cases} \frac{\Delta_3}{(2-n)^2} & \text{if } v < \frac{3}{4} \\ \frac{2(1-\frac{\Delta_3}{\Delta_3})-3n+n^2(2-v)}{n^2} & \text{otherwise} \end{cases} \]

\( c_7 \) is the relevant root of

\[ \sqrt{4c-1}(\sqrt{3(4c-k^2)}-2+k)(3-4v)+n(\sqrt{4c-1}(2-k)(3-4v)-(4c-1)(3-4v)^{3/2}) = 0. \] (A.1)

Table A.2 The green product’s optimal quality and price when \( Q = Q \) and \( q_t \leq n \)

<table>
<thead>
<tr>
<th>Condition</th>
<th>( q_g^* )</th>
<th>( p_g^* )</th>
<th>Dominant Subproblem</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q \leq \frac{1-n}{1-q_t} ) ( k \geq q_t )</td>
<td>( q_t )</td>
<td>( q_t + v(q_t - q_g^*) )</td>
<td>( S_2 )</td>
</tr>
<tr>
<td>( k \leq q_t )</td>
<td>( \frac{k+q_t(1-2v)}{2(1-v)} + )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q \geq \frac{1-n}{1-q_t} ) ( k \geq q_t )</td>
<td>( q_t )</td>
<td>( q_t + v(q_t - q_g^*) )</td>
<td>( S_1 )</td>
</tr>
<tr>
<td>( k \leq q_t )</td>
<td>( \frac{k+q_t(1-2v)}{2(1-v)} + )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table A.3  The green product’s optimal quality and price when \( Q = \tilde{Q} \) and \( q_t \geq n \)

<table>
<thead>
<tr>
<th>Condition</th>
<th>( q_g^* )</th>
<th>( p_g^* )</th>
<th>Dominant Subproblem</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{Q} \geq \frac{1-n}{1-q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_0 \leq k \leq q_t ) ( k \geq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
<tr>
<td>( \frac{1-n(1+\sqrt{7})}{1-q_t} \leq \tilde{Q} \leq \frac{1-n}{1-q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_1 \leq k \leq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
<tr>
<td>( \frac{2}{2-q_t} \leq \tilde{Q} \leq \frac{1-n(1+\sqrt{7})}{1-q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_1 \leq k \leq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
<tr>
<td>( \frac{2n}{q_t} \leq \tilde{Q} \leq \frac{2}{2-q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_1 \leq k \leq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
<tr>
<td>( \frac{n}{q_t} \leq \tilde{Q} \leq \frac{2n}{q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_1 \leq k \leq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
<tr>
<td>( n \leq q_t \leq \frac{2n}{1+q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_1 \leq k \leq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
<tr>
<td>( \tilde{Q} \leq \frac{n}{q_t} )</td>
<td>( k \geq q_t ) ( q_t )</td>
<td>( k_1 \leq k \leq q_t ) ( \frac{k+q_t(1-2v)}{2(1-v)} )</td>
<td>( q_g^* + v(q_t - q_g^*) )</td>
</tr>
</tbody>
</table>

\( \tilde{k}_0 = 2(1-v)(1-1/Q) - q_t(1-2v), \ k_1 = q_t - \frac{2n}{Q}, \ k_3 = q_t - \frac{2n}{Q}, \ k_4 = 2(1-1/Q) - q_t \)

Appendix B: Non-homogeneous Product Valuation within each Consumer Segment

We also investigate what happens if consumers do not have homogeneous product valuation within each segment. In that case, however, the analysis becomes intractable and therefore we resort to numerical studies.

We confirm that our key results continue to hold even when consumers have heterogeneous product valuation. Specifically, strategic abandonment still occurs when \( k \) is low and \( c \) is sufficiently high. In that case, resource consumption, \( Q^* \), increases in \( n \). Moreover, the firm may lower resource utilization (i.e., \( q_g^* \) increases) when production efficiency improves (i.e., \( k \) decreases). We present some of the numerical results below.

For example, consider \( q_t = 0.7, \ n = 0.3, \ v_t = \begin{cases} 1 \text{ with probability } \alpha \end{cases} \text{ and } v_g = \begin{cases} 0.15 \text{ with probability } 0.8 \\ 0.9 \text{ with probability } 1 - \alpha \end{cases} \). Note that these are the same parametric values for Figure 1, except that \( v_t \)
and $v_g$ now take multiple values. Figure 5 shows the result for $\alpha = 0.5$. Figure 5(a) shows that, similar to Figure 1, strategic abandonment occurs when $k$ is low and $c$ is sufficiently, but not overly, high. Also, similar to Figure 2, Figure 5(b) shows that the firm may lower resource utilization (i.e., $q_g^*$ increases) when production efficiency improves (i.e., $k$ decrease).

![Figure 5](image)

(a) Firm’s demand fulfillment pattern.  
(b) Optimal green product quality ($q_g^*$) when $c = 0.29$.

**Figure 5** The firm’s optimal decisions when $q_t = 0.7$, $n = 0.3$, $\alpha = 0.5$.

With the same $q_t = 0.7$, Figure 6 shows $Q^*$ for different $\alpha$, hence $v_1$, values when $c = 0.288$ and $k = 0.15$. Notice that with those parameters, strategic abandonment happens when $n < 0.65$; in that case, Figure 6 shows that $Q^*$ increases in $n$.

![Figure 6](image)

**Figure 6** $Q^*$ when $k = 0.15$, $q_t = 0.7$, $c = 0.288$ $v_t = 1$ with probability $\alpha$
As an additional example, Figure 7 considers $q_t = 0.7, n = 0.3, v_t = 1$ or $0.85$ with equal probabilities, and $v_g = 0.25$ or $0.15$ with equal probabilities.

Figure 7 The firm’s optimal decisions when $q_t = 0.7, n = 0.3, v_t = 1$ or $0.85$ with equal probabilities and $v_g = 0.25$ or $0.15$ with equal probabilities.

Appendix C: Proofs in §4.

We first provide the explicit expressions for the thresholds identified in Proposition 1:

\[
k_1 := c/(1-q_t), \tag{C.1}
\]
\[
k_2 := c/q_t - 1 - v \left\lfloor \frac{1}{1-q_t} \left[ \frac{(q_t-n)^+}{1-n} \right]^2 - (1-q_t)^2 \right\rfloor, \tag{C.2}
\]
\[
k_3 := c/q_t - \left\lfloor \frac{1}{1-q_t} \left( 1 + \sqrt{\frac{q_t}{1-n}} \right)^2 - 1 \right\rfloor (1-q_t), \tag{C.3}
\]
\[
k_4 := \begin{cases} 
1 - \frac{2(1-q_t)}{(1-n)^2} + \frac{1-q_t+n(1-v)(q_t+c}/q_t}{1-n} - \frac{2(1-q_t)}{1-n} \sqrt{\frac{q_t-n}{(1-q_t)(1-n)}} \left( \frac{c}{1-q_t} - \frac{1-v}{1-n} \right) + \frac{v}{(1-n)^2}, & \text{if } c \geq q_t - v q_t^2, \\
1 - \frac{2n(1-v)(1-q_t)}{(1-n)^2} + \frac{2n(1-v)(1-q_t)}{1-n} \sqrt{\frac{c}{1-q_t}} - \frac{n}{(1-n)^2}, & \text{if } c \leq q_t - v q_t^2,
\end{cases} \tag{C.4}
\]
\[
c^* := \left( 1 - q_t \right) q_t + \left( 1 - v \right) - \frac{n^2}{(1-n)^2} (1-q_t)^2, \tag{C.5}
\]

where $k_3, k_4$ and $c^*$ are only defined for $q_t \geq n$.

**Proof of Lemma 1.** For any given $q_g$, the firm incurs an cost of $c + k(q_t - q_g)$ for each unit of raw material, including the material acquisition cost and production cost for green products. On the other hand, each unit of raw material can generate $1-q_t$ traditional products that can be sold at price $q_t$ and $q_t - q_g$ green products that can be sold *at most* for a price $q_g + v(q_t - q_g)$. That is, the maximum possible revenue generated...
by each unit of raw material is $(1 - q_t)q_t + (q_t - q_g)[q_g + v(q_t - q_g)]$. Therefore, the firm enters the market and acquire positive quantity of raw material if and only if $(1 - q_t)q_t + (q_t - q_g)[q_g + v(q_t - q_g)] \geq c + k(q_t - q_g)$, which immediately implies condition (2).

When the firm enters the market, the raw material quantities needed to exactly match the traditional and green segment’s demands are $\frac{1 - n}{1 - q_t}$ and $\frac{n}{q_t - q_g}$, respectively. Therefore, we must have

$$\min \left\{ \frac{1 - n}{1 - q_t}, \frac{n}{q_t - q_g} \right\} \leq Q \leq \max \left\{ \frac{1 - n}{1 - q_t}, \frac{n}{q_t - q_g} \right\}. \quad (C.6)$$

In fact, since the material is costly, the firm never acquires more than $\max \left\{ \frac{1 - n}{1 - q_t}, \frac{n}{q_t - q_g} \right\}$. To see the lower bound in (C.6), if the firm acquires raw material less than $\min \left\{ \frac{1 - n}{1 - q_t}, \frac{n}{q_t - q_g} \right\}$, neither green nor traditional consumer segment is completely fulfilled, so the firm can sell additional green products at price $q_g + v(q_t - q_g)$ to green consumers and additional traditional products at price $q_t$ to traditional consumers at a marginal cost of $c + k(q_t - q_g)$. Under condition (2), the firm thus has incentive to increase the raw material quantity at least to $\min \left\{ \frac{1 - n}{1 - q_t}, \frac{n}{q_t - q_g} \right\}$.

If $q_g \geq \frac{n}{1 - q_t}$, (C.6) reduces to $\frac{1 - n}{1 - q_t} \leq Q \leq \frac{n}{q_t - q_g}$, implying that all the traditional segment’s demand can be satisfied by traditional products (i.e., $Q(1 - q_t) \geq 1 - n$) while the green segment’s demand may not be fully satisfied by green products (i.e., $Q(q_t - q_g) \leq n$). Since the green consumers are indifferent between a green product priced at $q_g + v(q_t - q_g)$ and a traditional product priced at $q_t \geq q_g + v(q_t - q_g)$, it is thus most profitable for the firm to produce just enough excess traditional products to satisfy the unfulfilled green consumers, i.e., $Q(1 - q_t) + Q(q_t - q_g) = 1$, while maintaining the price of green products at $q_g + v(q_t - q_g)$. This precisely yields the first region in the lemma.

If $q_g \leq \frac{n}{1 - q_t}$, (C.6) reduces to $\frac{n}{q_t - q_g} \leq Q \leq \frac{1 - n}{1 - q_t}$, implying that all the green segment’s demand can be satisfied by green products (i.e., $Q(q_t - q_g) \geq n$) while the traditional segment’s demand may not be fully satisfied by traditional products (i.e., $Q(1 - q_t) \leq 1 - n$). Therefore, the firm can consider the following two pricing strategies:

- If the green product is priced at $p_g = q_g + v(q_t - q_g)$, the traditional consumer will not purchase the green product, suggesting that either $Q = \frac{n}{q_t - q_g}$, i.e., the firm only produces enough green products to fulfill all the green demands and leaves some traditional demands unsatisfied, or $Q = \frac{1 - n}{1 - q_t}$, i.e., the firm acquires sufficient raw material to produce enough traditional products to fulfill all the traditional demands. The former case corresponds to the second region. In the latter case, however, the firm can be better off by increasing the green product’s quality $q_g$ so that no excess green products are produced (i.e., $Q(q_t - q_g) = n$) and the price $p_g = q_g + v(q_t - q_g)$ is also increased. This strategy is exactly the special case of the first region identified above.

- If the green product is priced at $p_g = q_g$, the traditional consumers become indifferent between the traditional and green products. Therefore, the firm can either acquire sufficient raw material (i.e., $Q(1 - q_t) = 1 - n$) so as to satisfy all the traditional consumers with traditional products, or acquires just enough raw material and produce excess green products to satisfy the traditional demand that is not fulfilled by traditional products, i.e., $Q(1 - q_t) + Q(q_t - q_g) = 1$. However, the former strategy is never optimal because the firm can then increase the green product’s price to $q_g + v(q_t - q_g)$, which brings the firm’s optimal decision
into the first region identified above. This leaves the latter strategy as only viable option, which corresponds to the third region in the lemma.

**Lemma C.1 (Solution of \((S_1)\)).** The optimal solution to \((S_1)\) is given by

\[
q^*_q \begin{cases} t, & \text{if } k \geq k_1, \\
1 - \sqrt{(1 - q_t)^2 + \frac{c - k(1 - q_t)}{1 - v}}, & \text{if } k_1 \geq k \geq k_2, \\
\frac{(1 - v)(1 - q_t) + c k}{(1 - v)(1 - q_t) + c k + 1 - k}, & \text{if } k \leq k_2,
\end{cases}
\]

and

\[
\Pi_1(c, k, q_t, v, n) = \begin{cases} q_t - \frac{c}{1 - q_t}, & \text{if } k \geq k_1, \\
(1 - 2v)(1 - q_t) - 2\sqrt{1 - v}(1 - q_t)^2 + c - k(1 - q_t) + 1 - k, & \text{if } k_1 \geq k \geq k_2, \\
(1 - k)q_t - (1 - v)q_t^2 - c, & \text{if } k \leq k_2, q_t \leq n, \\
q_t - \frac{n(1 - v)}{1 - n}(1 - q_t) - \frac{c(1 - n)}{1 - q_t} - nk, & \text{if } k \leq k_2, q_t \geq n.
\end{cases}
\]

which is nonnegative under condition \((2)\).

**Lemma C.2 (Solution of \((S_2)\)).** If \(q_t \geq n\), subproblem \((S_2)\) is feasible with its optimal solution given by

\[
q^*_q \begin{cases} \frac{q_t}{1 - v_n}, & \text{if } c \leq c^*, \\
q_t - \sqrt{c(1 - q_t) / (1 - v)}, & \text{if } c \geq c^*,
\end{cases}
\]

and

\[
\Pi_2(c, k, q_t, v, n) = \begin{cases} q_t - \frac{n(1 - v)}{1 - n}(1 - q_t) - \frac{(1 - n)c}{1 - q_t} - nk, & \text{if } c \leq c^*, \\
n(1 - (1 - v)q_t) - \frac{c}{q_t} - k, & \text{if } c \geq q_t - q_t^2,
\end{cases}
\]

which is positive for \(c^* \leq c \leq 1 - q_t q_t + \frac{(q_n - q_t)^2}{(1 - q_t)^2} \leq q_t - q_t^2\) if \(q_t \geq k \geq (2v - 1)q_t\), or for \(c^* \leq c \leq (1 - k)q_t - (1 - v)q_t^2\) if \(k \leq (2v - 1)q_t\).

**Lemma C.3 (Solution of \((S_3)\)).** If \(q_t \geq n\), subproblem \((S_3)\) is feasible with its optimal solution given by

\[
q^*_q \begin{cases} \frac{q_t}{1 - v_n}, & \text{if } k \geq k^{(3)}, \\
1 - \sqrt{(1 - q_t)^2 + c - k(1 - q_t)} + \frac{1}{1 - q_t} - 1, & \text{if } k \leq k^{(3)},
\end{cases}
\]

where \(k^{(3)} := \frac{c}{1 - q_t} - \left[\frac{1}{(1 - v_n)^2} - 1\right] (1 - q_t)\), and

\[
\Pi_3(c, k, q_t, v, n) = \begin{cases} 1 - \frac{(1 - n)}{1 - q_t} - \frac{(1 - n)c}{1 - q_t} - nk, & \text{if } k \geq k^{(3)}, \\
2 - 2\sqrt{(1 - q_t)^2 + c - k(1 - q_t)} - q_t - k, & \text{if } k^{(3)} \geq k \geq \frac{c}{1 - q_t} - \frac{1 - (1 - q_t)^2}{1 - q_t}, \\
(1 - k)(1 - q_t) - (1 - q_t)^2 - c - k, & \text{if } k \leq \frac{c}{1 - q_t} - \frac{1 - (1 - q_t)^2}{1 - q_t}.
\end{cases}
\]

In particular, \(\Pi_3(c, k, q_t, v, n) \leq \Pi_2(c, k, q_t, v, n)\) when \(k \geq k^{(3)}\).

**Proof of Proposition 1.** This proof consists of the following two parts.

**Properties of the thresholds.** First, the objective function in \((2)\) is decreasing in \(k\), so is \(\bar{c}(k, q_t, v)\) as its optimal value. In particular, it is a simple quadratic function in \(q_t\), allowing us to explicitly obtain

\[
\bar{c}(k, q_t, v) = \begin{cases} (1 - q_t)q_t, & \text{if } k \geq q_t, \\
(1 - q_t)q_t + \frac{(q_n - q_t)^2}{(1 - v_n)^2}, & \text{if } q_t \geq k \geq (2v - 1)q_t, \\
1 - (1 - v)q_t - k, & \text{if } k \leq (2v - 1)q_t.
\end{cases}
\]
Thus, we must have
\[
 c \leq \bar{c}(0, q_t, v) = \begin{cases} 
 1 - \left(1 - \frac{1}{4(1-v)}\right)q_t & \text{if } v \leq 1/2 \\
 1 - (1-v)q_t/q_t & \text{if } v \geq 1/2.
\end{cases}
\] (C.14)

By definition (C.1)-(C.5), it is clear that \( c^* \) is independent of \( k \), and \( k_i \) is linearly increasing in \( c \) for \( i = 1, 2, 3 \). The definition of \( k_2 \) in (C.2) implies
\[
k_2 = \begin{cases} 
 \frac{c}{1-q_t} - \frac{1-v}{1-q_t} \left\{ 1 - (1-q_t)^2 \right\}, & \text{if } q_t \leq n, \\
 \frac{c}{1-q_t} - (1-q_t) \left\{ \frac{1}{(1-n)^2} - 1 \right\}, & \text{if } q_t \geq n,
\end{cases}
\]
immediately suggesting that \( k_1 > k_2 \). In particular, when \( q_t \geq n \), we have \( k_2(c^*, q_t, v, n) = q_t - (1-v)(1-q_t)\frac{2n}{1-n} \in [(2v-1)q_t, q_t]. \) Hence, by (C.13) and (C.5), \( \bar{c}(k_2(c^*, q_t, v, n), q_t, v) = (1-q_t)q_t + (1-v)\frac{n^2}{(1-n)^2} (1-q_t)^2 = c^* \), i.e., \( k_2 \) as a function of \( c, \bar{c} \) as a function of \( k \) and \( c = c^* \) intersect at the exactly one point in the \((k, c)\)-plane. Therefore,
\[
c^* \leq \bar{c}(0, q_t, v) \iff k_2(c^*, q_t, v, n) \geq 0 \iff q_t \geq \max \left\{ n, \frac{2(1-v)n}{1+(1-2v)n} \right\} = \Gamma_1.
\] (C.15)

and
\[
c^* \leq c \leq \bar{c} \Rightarrow k \leq k_2.
\] (C.16)

Furthermore, straightforward calculation reveals that
\[
k_2 - k_3 = \frac{1}{1-q_t} \left\{ \frac{1-q_t}{(1-n)^2} - v(1-q_t)^2 - (1-v) \left[ 1 - \frac{(q_t-n)^+}{1-n} \right]^2 \right\}
\]
\[
= \begin{cases} 
 \frac{1}{1-q_t} \left\{ \frac{1-q_t}{(1-n)^2} - 1 + v\left[ 1 - (1-q_t)^2 \right] \right\} > 0, & \text{if } q_t \leq n, \\
 v(1-q_t) \left\{ \frac{1}{(1-n)^2} - 1 \right\} > 0, & \text{if } q_t \geq n,
\end{cases}
\]
and
\[
k_3 - k_3 = (1-q_t) \left[ \left( \frac{1+\sqrt{vn}}{1-n} \right)^2 - \frac{1}{(1-n)^2} \right] = \sqrt{vn}(2+\sqrt{vn}) \left( \frac{1}{(1-n)^2} \right) > 0.
\]
That is, we have demonstrated that
\[
k_2 > k_3 > k_3. \] (C.17)

Finally, we examine the property of \( k_4 \) by denoting
\[
k_4 := 1 - \frac{2(1-q_t)}{(1-n)^2} + \frac{1-q_t + n(1-v)q_t + c/q_t}{1-n} - \frac{2(1-q_t)}{1-n} \sqrt{\frac{q_t-n}{(1-q_t)(1-n)}} \left( \frac{c}{(1-q_t)q_t} - \frac{1-vn}{1-n} \right) + \frac{vn^2}{(1-n)^2},
\]
\[
\tilde{k}_4 := 1 - \frac{(1-n)^2(1-q_t)}{(1-n)^2} + \frac{2n\sqrt{1-v}\sqrt{c-(1-q_t)q_t}}{1-n} - \frac{2(1-q_t)}{1-n} \sqrt{\frac{\sqrt{c-(1-q_t)q_t} - n\sqrt{1-v}}{1-q_t} \frac{n\sqrt{1-v}}{1-n}} \left( \frac{1}{(1-n)^2} \right) + \frac{vn^2}{(1-n)^2}.
\]
One can directly verify that \( \tilde{k}_4 = k_4 \) when \( c = q_t - vq_t^2 \).

- If \( v \geq 1/2 \), we claim that \( \tilde{k}_4 \) is decreasing in \( c \) for all \( c \leq \bar{c}(0, q_t, v) \) such that \( \tilde{k}_4 \geq 0 \). Indeed, we first notice that \( \tilde{k}_4 \geq 0 \) is equivalent to
\[
\sqrt{\frac{q_t-n}{(1-q_t)(1-n)^2} + \frac{vn^2}{(1-n)^2}} \leq \frac{1}{2} \left[ \frac{1-n+n(1-v)q_t}{1-q_t} - 1 + \frac{vn^2}{1-n} + ny \right],
\] (C.18)
where $y := \frac{\sqrt{\bar{v}}}{1-q_t} - \frac{1 - vy}{1 - q_t} - \frac{1 - (1-v)q_t}{1 - q_t}$ because $c \leq \bar{c}(0, q_t) = [1 - (1-v)q_t] q_t$. Then, direct calculation reveals
\[
\frac{\partial \bar{k}_4}{\partial c} = n \sqrt{\frac{q_t - n}{(1-q_t)(1-n)}} y + \frac{vy}{(1-n)^2} - \frac{q_t - n}{(1-q_t)(1-n)}.
\] (C.19)

Thus, $\frac{\partial \bar{k}_4}{\partial c} < 0$ follows from (C.18):
\[
\begin{align*}
&n \sqrt{\frac{q_t - n}{(1-q_t)(1-n)}} y + \frac{vy}{(1-n)^2} - \frac{q_t - n}{(1-q_t)(1-n)} \\
&\leq \frac{n}{2} \left[ \frac{1 - n + n(1-v)q_t}{1 - q_t} - \frac{vy}{(1-q_t)(1-n)} \right] + n \left( \frac{1 - (1-v)q_t}{1 - q_t} - \frac{vy}{(1-q_t)(1-n)} \right) \\
&= \frac{(n-2)(q_t - n) - n^2(1-q_t)}{2(1-n)(1-q_t)} < 0.
\end{align*}
\] (C.20)

- We now show that $k_4$ is first increasing and then decreasing in $c \in [c^*, q_t - vq_t^2]$. Indeed, direct calculation reveals
\[
\begin{align*}
\frac{\partial \bar{k}_4}{\partial c} &= \frac{1}{(1-n)(1-q_t)z} \left[ n \sqrt{1-v} - \frac{z - \frac{n\sqrt{\bar{v}}}{1-n}}{\sqrt{\left( \frac{z - \frac{n\sqrt{\bar{v}}}{1-n}}{1-n} \right)^2 + \frac{vy}{(1-n)^2}}} \right] \\
&= \frac{1}{(1-n)(1-q_t)z} \left[ \frac{\sqrt{(1-v)n^2} - (1 - (1-v)n^2) \left( \frac{z - \frac{n\sqrt{\bar{v}}}{1-n}}{1-n} \right)^2}{\sqrt{\left( \frac{z - \frac{n\sqrt{\bar{v}}}{1-n}}{1-n} \right)^2 + \frac{vy}{(1-n)^2}}} \right]
\end{align*}
\]
where $z := \frac{\sqrt{-v(1-q_t)n}}{1-q_t}$.

One can immediately verify that $\frac{\partial \bar{k}_4}{\partial c} = \frac{\partial \bar{k}_4}{\partial c} < 0$ at $c = q_t - vq_t^2$ according to the analysis above, and that $\frac{\partial \bar{k}_4}{\partial c}$ is positive for $z \in \left[ \frac{n\sqrt{\bar{v}}}{1-n}, \frac{n\sqrt{\bar{v}}}{1-n} \left( 1 + \frac{n\sqrt{\bar{v}}}{\sqrt{1-(1-v)n^2}} \right) \right]$ and becomes negative for $z \geq \frac{n\sqrt{\bar{v}}}{1-n} \left( 1 + \frac{n\sqrt{\bar{v}}}{\sqrt{1-(1-v)n^2}} \right)$. Since $z$ has the same monotonicity as $c$ and $c \geq c^*$ is equivalent to $z \geq \frac{n\sqrt{\bar{v}}}{1-n}$, we thus conclude that $k_4$ is non-linearly increasing first and then decreasing in $c \in [c^*, q_t - vq_t^2]$. In particular, $k_4$ reaches its maximum
\[
k_4 = 1 - \frac{1 + v^2}{(1-n)^2(1-q_t)} + \frac{2(1-q_t)}{1-n} \left[ \frac{(1-v)n^2}{1-n} \left( 1 + \frac{n\sqrt{\bar{v}}}{\sqrt{1-(1-v)n^2}} \right) - \frac{\sqrt{(1-v)n^2} + \frac{v\sqrt{\bar{v}}}{1-n}}{1-(1-v)n^2} \right] \\
= 1 - (1-q_t) \left( \frac{\sqrt{(1-v)n^2} + \frac{v\sqrt{\bar{v}}}{1-n}}{1-n} \right)^2,
\]
at $z = \frac{n\sqrt{1-v}}{1-n} \left( 1 + \frac{n\sqrt{\bar{v}}}{\sqrt{1-(1-v)n^2}} \right)$. Therefore,
\[
k_4 \geq 0 \iff q_t \geq 1 - \left( \frac{1 - n}{\sqrt{1-(1-v)n^2} + \sqrt{\bar{v}}} \right)^2 = \Gamma_2,
\] (C.21)
where $\Gamma_2 > \Gamma_1$ because direct calculation reveals
\[
\Gamma_2 - n = \frac{n(1-n)}{\left( \frac{\sqrt{1-(1-v)n^2 + \sqrt{\bar{v}}} }{1-(1-v)n^2 + \sqrt{\bar{v}}} \right)^2} \left[ vn + 1 - (1-v)n + 2\sqrt{v} \sqrt{1-(1-v)n^2} \right] > 0
\]
and
\[
\Gamma_2 - \frac{2(1-v)n}{1+(1-2v)n} = \frac{2(1-n)\sqrt{\bar{v}}}{(1+(1-2v)n) \left( \frac{\sqrt{1-(1-v)n^2 + \sqrt{\bar{v}}} }{1-(1-v)n^2 + \sqrt{\bar{v}}} \right)^2} \left[ \sqrt{1-(1-v)n^2} + \sqrt{\bar{v}} \right] > 0.
\]
To show that $k_4 \leq k_3$, we simply need to show that $k_4 \leq k_3$. Straightforward calculation then yields

$$k_3 - k_4 = (1 - q_t) \left[ \frac{(1 - v)n^2 - 2\sqrt{v}n}{(1 - n)^2} + z^2 - \frac{2n\sqrt{1 - v}}{1 - n}z + \frac{2}{1 - n} \sqrt{\left( z - \frac{n\sqrt{1 - v}}{1 - n} \right)^2 + \frac{vn^2}{(1 - n)^2}} \right]$$

$$= (1 - q_t) \left[ \left( \sqrt{\left( z - \frac{n\sqrt{1 - v}}{1 - n} \right)^2 + \frac{vn^2}{(1 - n)^2}} + \frac{1}{1 - n} \right)^2 - \left( \frac{1 + \sqrt{v}n}{1 - n} \right)^2 \right] \geq 0,$$

where the equality holds if and only if

$$z = \frac{\sqrt{c} - (1 - q_t)q}{1 - q_t} = \frac{n\sqrt{1 - v}}{1 - n}, \quad \Leftrightarrow \quad c = (1 - q_t)q + (1 - v)\frac{n^2}{(1 - n)^2}(1 - q_t)^2 = c^*, \text{ by (C.5).}$$

**Comparison of the subproblems.** We notice that if $q_t < n$, subproblem $(S_2)$ and $(S_3)$ are infeasible, and hence, the optimal solution to $(P)$ is given by that of $(S_1)$ characterized by Lemma C.1. In particular, $(k, c)$-region $\Omega_2$ and $\Omega$ disappear in this case. Thus, we restrict the subsequent proof to the case $q_t \geq n$, under which the optimal solution to $(P)$ is given by that of the subproblems $(S_1)$, $(S_2)$ and $(S_3)$ with the highest optimal values. Lemma C.1, C.2 and C.3 suggest the following detailed comparison:

- When $c \leq c^*$ and $k \geq k^{(3)}$, subproblem $(S_2)$ dominates $(S_3)$ by Lemma C.3. By (C.17), $k_2 \geq k^{(3)}$. For $k_2 \geq k \geq k^{(3)}$, we have the subproblems $(S_1)$ and $(S_2)$ coincide, i.e., $\Pi_1(c, k, q_t, v, n) = \Pi_2(c, k, q_t, v, n)$. If $k \geq k_2$, we have

$$\Pi_1(c, k, q_t, v, n) - \Pi_2(c, k, q_t, v, n) \geq (1 - 2v)(1 - q_t) - 2\sqrt{1 - v}(1 - q_t)^2 + c - k(1 - q_t) + 1 - k \left[ q_t - \frac{n^2(1 - v)}{1 - n} + c(1 - n) \right]$$

$$= \frac{1 - n}{1 - q_t} \left[ 1 - q_t \sqrt{1 - v} - (1 - q_t)^2 + c - k(1 - q_t) \right]^2 \geq 0,$$

where the equality holds if and only if $k = k_2$. Therefore, when $k \geq k_2$, the subproblem $(S_1)$ dominates $(S_2)$.

- When $c \leq c^*$ and $k \leq k^{(3)}$, since $k^{(3)} < k_3$ by (C.17), we have the subproblems $(S_1)$ and $(S_2)$ coincide, i.e., $\Pi_1(c, k, q_t, v, n) = \Pi_2(c, k, q_t, v, n)$, in this region. We next notice that the subproblem $(S_3)$ must be dominated for $k \leq \frac{c}{1 - q_t} - \frac{1 - (1 - q_t)^2}{1 - q_t}$, in which case

$$\Pi_3(c, k, q_t, v, n) = (k + 1)(1 - q_t) - (1 - q_t)^2 - c - k \leq c - 1 + (1 - q_t)^2 + (1 - q_t) - (1 - q_t)^2 - c - k = -q_t - k < 0. \quad (C.22)$$

For $k^{(3)} \geq k \geq \frac{c}{1 - q_t} - \frac{1 - (1 - q_t)^2}{1 - q_t}$, we have

$$\Pi_2(c, k, q_t, v, n) - \Pi_3(c, k, q_t, v, n)$$

$$= q_t - \frac{n^2(1 - v)}{1 - n} + (1 - q_t) - \frac{(1 - n)c}{1 - q_t} - nk - 2 + 2\sqrt{(1 - q_t)^2 + c - k(1 - q_t)} + q_t + k$$

$$= \frac{1 - n}{1 - q_t} \left[ \frac{vn^2(1 - q_t)^2}{(1 - n)^2} - \left( \sqrt{(1 - q_t)^2 + c - k(1 - q_t)} - \frac{1 - q_t}{1 - n} \right)^2 \right].$$

Since when $k \leq k^{(3)}$, we have $1 - \sqrt{(1 - q_t)^2 + c - k(1 - q_t)} \leq \frac{q_t - n}{1 - n}$, namely $\sqrt{(1 - q_t)^2 + c - k(1 - q_t)} \geq \frac{1 - q_t}{1 - n}$. Therefore, $\Pi_2(c, k, q_t, v, n) \leq \Pi_3(c, k, q_t, v, n)$ if and only if $\sqrt{(1 - q_t)^2 + c - k(1 - q_t)} - \frac{1 - q_t}{1 - n} \geq \frac{q_t - n}{1 - n}$, or equivalently,

$$k \leq \frac{c}{1 - q_t} - \left( \frac{1 + \sqrt{v}n}{1 - n} \right)^2 \left( 1 - q_t \right) = k_3 < k^{(3)}. \quad (C.23)$$
When \( c^* \leq c \leq \tilde{c} \), (C.16) implies that \( k \leq k_2 \), therefore we must have
\[
\Pi(c, k, q, v, n) = q_t - \frac{n^2(1 - v)}{1 - n}(1 - q_v) - \frac{c(1 - n)}{1 - q_v} - nk
\]
\[
\leq \Pi_2(c, k, q, v, n) = \left\{ \begin{array}{ll}
q_t - 2\sqrt{1 - v}\sqrt{c - (1 - q_t)q_v - k} & \text{if } c^* \leq q_t - vq_t^2,
q_t - 2\sqrt{1 - v}\sqrt{c - (1 - q_t)q_v - k} & \text{if } c \geq q_t - vq_t^2,
\end{array} \right.
\]
which is nonnegative by Lemma C.2 and (C.13). Namely, subproblem \((k)\) which is equivalent to
\[
\Pi(c, k, q, v, n) = 2 - 2\sqrt{(1 - q_t)^2 + c - k(1 - q_t)} - q_t - k - n\left\{ q_t - 2\sqrt{1 - v}\sqrt{c - (1 - q_t)q_v - k} \right\}
\]
reduces to
\[
(1 - n)x + \frac{1 + n(1 - v)q_t + c/q_t}{1 - n} = 4\left\{ \begin{array}{ll}
\frac{c}{(1 - q_t)q_t} - 1 - vn & \text{if } c \geq q_t - vq_t^2,
\frac{c}{(1 - q_t)q_t} - 1 - vn & \text{if } c \geq q_t - vq_t^2,
\end{array} \right.
\]
which is equivalent to \( k \leq \bar{k}_4 \).
Proof of Corollary 1 and 2. The expressions of \( q^*_g \) and \( Q^* \) given in Table 3 immediately indicate that they are monotonically non-increasing in \( c \) and non-decreasing in \( k \) in each individual regions \( \Omega_i \) for \( i \in \{0, 1, 2, 3, 12\} \). Thus, we just need to check their monotonicity when crossing the boundaries between regions. It is straightforward to verify that \( q^*_g \) and \( Q^* = \frac{1}{1-q^*_g} \) are continuous at boundaries \( k = k^*_j \) for \( j = 1, 2 \) and \( c = c^* \). Direct calculation reveals that \( q^*_g|_{k^*_1, k^*_2} = 1 - \frac{1+\sqrt{n}}{1-n} (1-q^*_t) < \frac{n-v}{1-n} = q^*_g|_{k^*_2, k^*_3} \) which suggests that \( q^*_g \) jump downward when \( c \) increases from region \( \Omega_{12} \) to region \( \Omega_3 \) across the boundary \( k = k_3 \), and that they jump upward when \( k \) increases from region \( \Omega_3 \) to region \( \Omega_{12} \) across the boundary \( k = k_3 \).

Finally, for \( c \geq q^*_t - vq^2 \), we must have \( q^*_g|_{k^*_1, k^*_2} = 0 \leq q^*_g|_{k^*_3, k^*_4} \), and

\[
Q^*|_{k^*_1, k^*_2} = Q^*|_{k^*_3, k^*_4} = \sqrt{\frac{1}{1-n}} \left[ \frac{1}{1-n} + \sqrt{\left( \frac{\sqrt{c - (1-q^*_t)q^*_t}}{1-q^*_t} - \frac{n\sqrt{1-v}}{1-n} \right)^2 + \frac{vn^2}{(1-n)^2}} \right],
\]

\[\text{and} \quad Q^*|_{k^*_1, k^*_2} = q^*_t - \sqrt{\frac{c - (1-q^*_t)q^*_t}{1-v}} , \quad Q^*|_{k^*_3, k^*_4} = \frac{n\sqrt{1-v}}{\sqrt{c - (1-q^*_t)q^*_t}}.\]

Thus, \( q^*_g|_{k^*_1, k^*_2} \geq q^*_g|_{k^*_3, k^*_4} \) is equivalent to

\[
\frac{\sqrt{c-(1-q^*_t)q^*_t}}{1-q^*_t} \geq \frac{2n\sqrt{1-v}}{1-n} \Leftrightarrow c \geq (1-q^*_t)q^*_t + 4(1-v)\frac{n^2}{(1-n)^2}(1-q^*_t)^2,
\]

while

\[
Q^*|_{k^*_1, k^*_2} - Q^*|_{k^*_3, k^*_4} = \sqrt{\frac{1}{1-n}} \left[ \frac{1}{1-n} + \sqrt{\left( \frac{\sqrt{c - (1-q^*_t)q^*_t}}{1-q^*_t} - \frac{n\sqrt{1-v}}{1-n} \right)^2 + \frac{vn^2}{(1-n)^2}} \right].
\]

has the same sign as \( \partial k^*_3/\partial c \) which is shown to be positive first and then negative in the proof of Proposition 1. Therefore, \( q^*_g \) jumps downward as \( c \) or \( k \) increases from region \( \Omega_3 \) to region \( \Omega_2 \) by crossing the boundary \( k = k_4 \), when \( c \geq \min \left\{ q^*_t - vq^2, (1-q^*_t)q^*_t + 4(1-v)\frac{n^2}{(1-n)^2}(1-q^*_t)^2 \right\} > c^* \). On the other hand, \( Q^* \) can only have downward jumps as \( c \) increases from \( \Omega_2 \) to \( \Omega_3 \) and then from \( \Omega_3 \) back to \( \Omega_2 \); \( Q^* \) jumps upward as \( k \) increases from \( \Omega_3 \) to \( \Omega_2 \) by crossing the increasing section of \( k_4 \) but downward when \( k \) crosses the decreasing section of \( k_4 \).  \( \square \)
Proof of Table 4. The monotonicity of the three environmental performance metrics follows from direct examination of the expressions for the optimal \( q_g^* \) and \( Q^* \) obtained in Table 3:

1. In region \( \Omega_0 \), \( q_g^* = q_t \) and \( Q^* = 1/(1-q_t) \) are both independent of \( n \) and \( v \), so is \( \Pi^* = q_t - \frac{c}{1-q_t} \), leading to the first row of Table 4.

2. In region \( \Omega_1 \), \( q_g^* = 1 - \sqrt{(1-q_t^2 + c-k(1-q_t))/(1-v)} \) is independent of \( n \) but non-increasing in \( v \), so is \( Q^* = 1/(1-q_g^*) \). Therefore, \( U^* = 1 - q_g^* \) is independent of \( n \) and non-decreasing in \( v \); \( W^* = 1/(1-q_g^*) - 1 \) is independent of \( n \) and non-increasing in \( v \). By (C.8), we have

\[
\Pi^* = (1-2v)(1-q_t) - 2\sqrt{1-v}\sqrt{(1-v)(1-q_t)^2} + c - k(1-q_t) + 1 - k,
\]

which immediately implies that \( \Pi^* \) is independent of \( n \) and has

\[
\frac{\partial \Pi^*}{\partial v} = -2(1-q_t) + \frac{\sqrt{1-v}}{(1-v)(1-q_t)^2 + c - k(1-q_t)}(1-q_t)^2 + \frac{(1-v)(1-q_t)^2 + c - k(1-q_t)}{\sqrt{1-v}} \geq -2(1-q_t) + 2(1-q_t) = 0
\]

Hence, the second row of Table 4 is obtained.

3. In region \( \Omega_2 \), \( q_g^* = q_t - n/(n-q_t) \) is independent of \( n \) but non-increasing in \( v \). Therefore, \( Q^* = n/(n-q_t) \) is increasing in \( n \) but non-increasing in \( v \); \( U^* = 1 - q_g^* \) is independent of \( n \) but non-decreasing in \( v \); \( W^* = n(n/(n-q_t) - 1) \) is increasing in \( n \) and non-increasing in \( v \). In this region, \( \Pi^* \) is given by (C.10), which is obviously increasing in \( n \) and \( v \). Hence, the third row of Table 4 is obtained.

4. In region \( \Omega_{12} \), \( q_g^* = 1 - \frac{1-n}{1-v} \) is decreasing in \( n \) but independent of \( v \), so is \( Q^* = 1/(1-q_g^*) \). Therefore, \( U^* = 1 - q_g^* \) is increasing in \( n \) and independent of \( v \); \( Q^* = 1/(1-q_g^*) \) is decreasing in \( n \) and independent of \( v \). In this region, (C.8) and (C.10) imply that \( \Pi^* = q_t - \frac{n^2(1-v)}{(1-n)^2} - \frac{(1-n)v}{1-q_t} - nk \), which is obviously increasing in \( v \) and has \( \frac{\partial \Pi^*}{\partial n} = \frac{c}{1-q_t} - (1-v)(1-q_t) \left[ \frac{1}{(1-n)^2} - 1 \right] - k \geq 0 \), because \( k \leq k_2 \leq \frac{c}{1-q_t} - (1-v)(1-q_t) \left[ \frac{1}{(1-n)^2} - 1 \right] \) by definition. Hence, the fourth row of Table 4 is obtained.

5. In region \( \Omega_0 \), \( q_g^* = q_t \) and \( Q^* = 1/(1-q_t) \) are both independent of \( n \) and \( v \), so is \( \Pi^* \) by (C.12) (for \( k \leq k_3 \leq k(3) \)), leading to the last row of Table 4. \( \square \)
Designing Sustainable Products under Co-Production Technology

Appendix D: Proofs in Appendix C

Proof of Lemma C.1. We first notice that the objective function in (S_1) can be rewritten as
\[
\left(1 - q_t\right)q_t + (q_t - q_g)[q_g + v(q_t - q_g) - k] - c
\]
which must be nonnegative for some \(q_g\) under condition (2), immediately suggesting \(\Pi_1(c, k, q_t, v, n) \geq 0\).

Straightforward calculation yields the first derivative of the objective function in (S_1) with respect to \(q_g\):
\[
\frac{\partial}{\partial q_g} \left\{ \left(1 - q_t\right)q_t + \left(\frac{q_t - q_g}{1 - q_g}\right) [q_g + v(q_t - q_g)] - \frac{c}{1 - q_g} - k \left(\frac{q_t - q_g}{1 - q_g}\right) \right\} = (1 - v) \left[ 1 - \left(\frac{1 - q_t}{1 - q_g}\right)^2 \right] + \frac{k(1 - q_t) - c}{(1 - q_g)^2},
\]
which is nonnegative for all \(q_g \in [0, q_t]\) if \(k(1 - q_t)c\), or equivalently \(k \geq k_1\) and is decreasing in \(q_g \in [0, q_t]\) if \(k \leq k_1\). Therefore, when \(k \geq k_1\), the optimal solution to (S_1) is achieved at the upper bound \(q_t\); when \(k \leq k_1\), the optimal solution to (S_1) is given by \(\max \left\{ 1 - \sqrt{\left(1 - q_t\right)^2 + \frac{c - k\left(1 - q_t\right)^2}{1 - v}}, \frac{q_t - q_g}{1 - q_g} \right\}\), from which the threshold \(k_2\) and the solution in (C.7) follow. Plugging (C.7) into the objective function of (S_1) immediately yields (C.8). □

Proof of Lemma C.2. The objective function of (S_2) can be rewritten as
\[
- n \left\{ \frac{c - (1 - q_t)q_t}{q_t - q_g} + (1 - v)(q_t - q_g) + k - q_t \right\},
\]
which is increasing in \(q_g\) if \(c \leq (1 - q_t)q_t\) and is concave in \(q_g\) if \(c > (1 - q_t)q_t\). In the former case, the optimal solution to (S_2) is achieved at the upper bound \(\frac{c - q_t}{1 - v}\). In the latter case, the derivative of the objective function in (S_2) with respect to \(q_g\) can be calculated as
\[
- n \left( 1 - v - \frac{c - (1 - q_t)q_t}{(q_t - q_g)^2} \right),
\]
which implies that the optimal solution to (S_2) is given by
\[
\min \left\{ \left( q_t - \sqrt{\frac{c - (1 - q_t)q_t}{1 - v}} \right)^+ + \frac{q_t - n}{1 - n} \right\} = \left\{ \begin{array}{ll} \frac{c - q_t}{1 - v} & \text{if } c \leq c^*, \\ q_t - \sqrt{\frac{c - (1 - q_t)q_t}{1 - v}} & \text{if } c > c^*, \end{array} \right.
\]
and hence the optimal solution (C.9) follows. Plugging (C.9) into the objective function of (S_2) immediately yields (C.10).

Finally, it is straightforward to verify that
- if \(q_t \geq k \geq (2v - 1)q_t\) and \(c^* \leq c \leq (1 - q_t)q_t + \frac{(q_t - k)^2}{4(1 - v)} \leq q_t - vq_t^2\),
  \[\Pi_2(c, k, q_t, v, n) = n \left\{ q_t - 2\sqrt{1 - v}\sqrt{c - (1 - q_t)q_t - k} \right\} \geq n \left\{ q_t - 2\sqrt{1 - v}\sqrt{\frac{(q_t - k)^2}{4(1 - v)} - k} \right\} = 0;\]
- if \(k \leq (2v - 1)q_t\) and \(c^* \leq c \leq (1 - k)q_t - (1 - v)q_t^2\),
  \[\Pi_2(c, k, q_t, v, n) \geq n \left\{ 1 - (1 - v)q_t - \frac{c}{q_t} - k \right\} \geq n \left\{ 1 - (1 - v)q_t - \frac{(1 - k)q_t - (1 - v)q_t^2}{q_t} - k \right\} = 0. \ □\]
Proof of Lemma C.3. The objective function of (S3) can be rewritten as
\[ q_\theta = \frac{c - k(1 - q_\theta) + (1 - q_\theta)^2}{1 - q_\theta} - q_e - k + 1, \]
which is increasing in \( q_\theta \) if \( c - k(1 - q_\theta) - (1 - q_\theta)^2 \leq 0 \) and is concave in \( q_\theta \) otherwise. In the former case, the optimal solution to (S3) is achieved at the upper bound \( \frac{q_e - n}{1 - n} \). In the latter case, the derivative of the objective function in (S3) with respect to \( q_\theta \) can be calculated as
\[ 1 - \frac{(1 - q_\theta)^2 + c - k(1 - q_\theta)}{(1 - q_\theta)^2}, \]
which implies that the optimal solution to (S3) is given by
\[ \min \left\{ \left( 1 - \sqrt{(1 - q_t)^2 + c - k(1 - q_t)} \right)^+ + \frac{q_e - n}{1 - n}, \frac{q_e - n}{1 - n} \right\} = \left\{ \left( 1 - \sqrt{(1 - q_t)^2 + c - k(1 - q_t)} \right)^+ + \frac{q_e - n}{1 - n} \right\}, \]
and hence the optimal solution (C.11) follows. Plugging (C.11) into the objective function of (S3) immediately yields (C.12).

Finally, to see that \( \Pi_3(c, k, q_t, v, n) \leq \Pi_2(c, k, q_t, v, n) \) when \( k \geq k^{(3)} \), we notice that
\[ \Pi_2(c, k, q_t, v, n) \geq q_t - \frac{n(1 - v)}{1 - n}(1 - q_t) - \frac{(1 - n)c}{1 - q_t} - nk \]
\[ \geq 1 - \left( \frac{1}{1 - n} - n \right)(1 - q_t) - \frac{c(1 - n)}{1 - q_t} - nk = \Pi_3(c, k, q_t, v, n). \]

Appendix E: Proofs in Appendix A

Proof of Proposition A.1. As Proposition 1 characterizes the firm’s optimal decisions \( q_\theta^*, p_\theta^*, p_\theta^* \) and \( Q^* \) as well as identifies the dominant subproblems for any given \( q_t \). We thus just need to further optimize the firm’s optimal profit function in (P) over \( q_t \). In the following proof, we introduce an additional subscript \( b \) to indicate the optimal solutions in the case of exogenous \( q_t \) obtained in Proposition 1.

Region \( \Omega_3 \). Proposition 1 and Lemma C.1 suggest the firm’s optimal profit to be
\[ \Pi_1(q_t) = (1 - 2v)(1 - q_t) - 2\sqrt{1 - v}(1 - q_t)^2 + c - k(1 - q_t) + 1 - k. \]
Direct calculation reveals that
\[ \frac{\partial^2 \Pi_1(q_t)}{\partial q_t^2} = \frac{-4(1 - v)(1 - q_t)^2 + c - k(1 - q_t)}{2(1 - v)^2}, \]
Assume for now that \( c > \frac{k^2}{4(1 - v)} \) so \( \Pi_1(q_t) \) is concave in \( q_t \). This allows us to apply first-order condition to obtain the optimal \( q_t \). We will show later that \( c > \frac{k^2}{4(1 - v)} \) indeed holds. The first order condition, \( \partial \Pi_1(q_t)/\partial q_t = 0 \), immediately yields \( q_t^* = 1 + \frac{(2v - 1)\sqrt{4(1 - v) - k^2}}{2(1 - v)} \) and the optimal green product quality \( q_t^* = 1 - \sqrt{(1 - q_t^*)^2 + \frac{c - k(1 - q_t^*)}{4(1 - v)}} = 1 - \sqrt{\frac{4(1 - v) - k^2}{4(1 - v)}}. \) Remind that \( q_t^* \) and \( q_t^* \) are feasible in region \( \Omega_3 \) only if \( c \leq \hat{c}(k, q_t^*, v) \) and \( k_1(c, q_t^*, v, n) \leq k \leq k_2(c, q_t^*, v, n) \) by Proposition 1, which lead to
\[ c \leq \hat{c}(k, q_t^*, v) \iff c \leq \frac{1 - k + k^2 - v}{3 - 4v}, \quad k \geq k_2(c, q_t^*, v, n) \iff c \leq \frac{k^2\Delta_3}{2(1 - v)}^2, \quad k \leq k_1(c, q_t^*, v, n) \iff c \geq k^2. \]
It can be shown that \( \frac{k^2\Delta_3}{2(1 - v)^2} \) decreases in \( v \) and \( \frac{k^2\Delta_3}{2(1 - v)^2} = k^2 \) for \( v = \frac{3}{4} \). Hence region \( \Omega_4 \) does not exist for \( v \geq \frac{3}{4} \). Also we can show that \( k^2 > \frac{k^2}{4(1 - v)} \) for \( v < \frac{3}{4} \). Combining with the requirement \( c \geq k^2 \), it implies that \( c > \frac{k^2}{4(1 - v)} \) holds and \( \Pi_1(q_t) \) is concave in \( q_t \) whenever \( q_t^* = 1 + \frac{(2v - 1)\sqrt{4(1 - v) - k^2}}{2(1 - v)} \) is feasible.
Proposition 1 and Lemma C.1 suggest the firm’s optimal profit to be

$$\Pi_0(q_t) = q_t - \frac{c}{1-q_t},$$

which is obviously concave in \(q_t\). The first order condition yields \(q_t^* = q_t^* = 1 - \sqrt{c}\). Remind that \(q_t^* + q_t^*\) are feasible in region \(\Omega_0\) only if \(c \leq \bar{c}(k, q_t^*, v)\) and \(k \geq k_1(c, q_t^*, v, n)\) by Proposition 1, leading to

$$c \leq \bar{c}(k, q_t^*, v) \Rightarrow c \leq \frac{1}{4}, \quad \text{and} \quad k \geq k_1(c, q_t^*, v, n) \Rightarrow c \leq k^2.$$

Region \(\Omega_{12}\) By Proposition 1, subproblems \((S_1)\) and \((S_2)\) coincide. Therefore, Lemma C.2 suggest the firm’s optimal profit to be

$$\Pi_{12}(q_t) = q_t - n^2(1-v) \frac{(1-q_t)}{1-n} - (1-n)c - nk,$$

which is obviously concave in \(q_t\). Therefore, the first order condition yields \(q_t^* = 1 - (1-n)\sqrt{\frac{c}{1-n+n^2(1-v)}}, q_t^* = 1 - \sqrt{\frac{c}{1-n+n^2(1-v)}}\). Remind that those new \(q_t^*\) and \(q_t^*\) are feasible in region \(\Omega_{12}\), which requires \(c \leq c^*(q_t^*, v, n), k_3(c, q_t^*, v, n) \leq k \leq k_2(c, q_t^*, v, n) \) and \(q_t^* \geq \Gamma_1\) by Proposition 1, leading to

$$c \leq c^*(q_t^*, v, n) \Leftrightarrow c \leq \frac{\Delta_3}{(2-n)^2}, \quad q_t^* \geq \Gamma_1 \Leftrightarrow k < \frac{1 - 2n(1-v)}{2-n}, \quad k \leq k_2(c, q_t^*, v, n) \Leftrightarrow c \geq \frac{k^2 \Delta_3}{(1-2n(1-v))^2},$$

and

$$k \geq k_3(c, q_t^*, v, n) \Leftrightarrow c \leq \frac{k^2(1-5n+n^2(9+2v)-4n^3(2-v)+4n^4(1-v)+2(1-2n)n\sqrt{3}\Delta_3)}{(1-4n+4n^2(1-v)^2)}.$$

Region \(\Omega_2\) Proposition 1 and Lemma C.2 suggest the firm’s optimal profit to be

$$\Pi_2(q_t) = \begin{cases} 
\frac{n}{q_t} \left( q_t - 2\sqrt{1-v} \sqrt{c - (1-q_t)q_t} - k \right), & \text{if } c^* \leq c \leq q_t - vq_t^2, \\
\left( 1 - (1-v)q_t - \frac{c}{q_t} - k \right), & \text{if } c \geq q_t - vq_t^2,
\end{cases}$$

which implies that

$$\frac{\partial^2 \Pi_2(q_t)}{\partial q_t^2} = \begin{cases} 
\frac{n}{q_t} \left( c - (1-q_t)q_t \right) \frac{3}{3}, & \text{if } c^* \leq c \leq q_t - vq_t^2, \\
\frac{c - (1-q_t)q_t}{q_t^2}, & \text{if } c \geq q_t - vq_t^2.
\end{cases}$$

 Apparently \(\Pi_2(q_t)\) is concave in \(q_t\) for \(c > 1/4\). In the following, first we derive the optimal \(q_t\) when \(c > 1/4\) and we will show that region \(\Omega_2\) is irrelevant for \(c < 1/4\).

For \(c > 1/4\), the first-order condition yields \(q_t^* = \frac{1}{2}(1 + \sqrt{\frac{4c - 1}{3-4v}})\) and \(q_t^* = \frac{1}{2}(1 - \sqrt{\frac{4c - 1}{3-4v}})\). Apparently this solution is only feasible for \(v < \frac{3}{4}\).

(1) When \(v < \frac{3}{4}\), we need to confirm that \(c > \frac{1}{4}\) and \(\frac{c - (1-q_t)q_t}{1-v} > 0\) hold for \(q_t^* = \frac{1}{2}(1 + \sqrt{\frac{4c - 1}{3-4v}})\) and \(q_t^* = \frac{1}{2}(1 - \sqrt{\frac{4c - 1}{3-4v}})\). First, by Proposition 1 region \(\Omega_2\) is feasible only \(c \geq c^*\). With \(q_t^* = \frac{1}{2}(1 - \sqrt{\frac{4c - 1}{3-4v}})\) it implies

$$c \geq \frac{1 - n + n^2(1-v) (2-n)^2}{(2-n)^2} > \frac{1}{4},$$

and hence \(c > \frac{1}{4}\) is immediate. Notice that the above inequality also implies that \(q_t^*\) is irrelevant for \(c < \frac{1}{4}\).

Second, it is straightforward that

$$\frac{c -(1-q_t^*)q_t}{1-v} = \frac{4c - 1}{3-4v} > 0$$

when \(v < \frac{3}{4}\) and \(c > \frac{1}{4}\). Remind that \(q_t^*\) and \(q_t^*\) are feasible in region \(\Omega_2\) only if \(q_t^* \geq \Gamma_1(n, v), c \geq c^*(q_t^*, v, n), c \leq \tilde{c}(q_t^*, v, n), k \geq k_1(c, q_t^*, v, n)\) by Proposition 1, leading to

$$q_t^* \geq \Gamma_1(n, v) \Leftrightarrow k \leq \frac{1-2n(1-v)}{2-n}, \quad c \geq c^*(q_t^*, v, n) \Leftrightarrow c \geq \frac{\Delta_3}{(2-n)^2},$$
where $c_7$ is the relevant root of (A.1). Moreover, it can be shown that $c \in \left[\frac{\sqrt{3}}{4}, \frac{\sqrt{3}+k^2-v}{3-4v} \right]$ is non-empty and hence $q^*_t$ is relevant implies $k < \frac{1}{2}$. So we conclude that $q^*_t = \frac{1}{2}(1+\sqrt{\frac{4k-1}{3-4v}})$ and $q^*_g = \frac{1}{2}(1-\sqrt{\frac{4k-1}{3-4v}})$ for region $\Omega_2$ when $v < \frac{3}{4}$ and $k < \frac{1}{2}$.

(2) When $v \geq \frac{3}{4}$, $\frac{\partial^2 \Pi_2(q_t)}{\partial q_t^2} \bigg|_{q_t^*} = n(1-\frac{\sqrt{1-k^2}}{v})$. Assume now that $c \geq 1 - v$ so that $\frac{\partial^2 \Pi_2(q_t)}{\partial q_t^2} \bigg|_{q_t^*} \geq 0$ and hence $q^*_t = 1$ and $q^*_g = 0$. In that case,

$$\Pi_2(q_t = 1, q_g = 0) > \Pi_{12}(q^*_t, q^*_g) \Leftrightarrow c > c_6 = \frac{2(1-(1-n)\sqrt{3})-3n+n^2(2-v)}{n^2} > 1-v$$

and hence the assumption $c \geq 1 - v$ is satisfied. Also, it is straightforward that $c \geq 1 - v \geq \frac{1}{2}$ for $v \geq \frac{3}{4}$ so again $q^*_t = 1$ is irrelevant for $c < \frac{1}{4}$. Moreover, we require $c \leq \tilde{c}(q^*_t, v, n)$ by Proposition 1, leading to $c \leq v-k$.

We also require that $v-k > \frac{1}{4}$ otherwise the entire $\Omega_2$ region is dominated by $\Omega_0$ region, which implies $v > k + \frac{1}{4}$. So we conclude that $q^*_t = 1$ and $q^*_g = 0$ for region $\Omega_2$ when $v \geq \max\left(\frac{1}{4}, k + \frac{1}{4}\right)$. Moreover, region $\Omega_2$ does not exist for $k \geq \frac{1}{2}$ and $v \leq k + \frac{1}{4}$. In fact, only region $\Omega_0$ exists in that case. In sum, the above discussion leads to

$$c_4 = \begin{cases} 
\frac{1-k^2+2v}{3-4v} & \text{if } k \leq \frac{1}{2} \text{ and } v < \frac{3}{4} \\
v-k & \text{if } v \geq \max\left(\frac{3}{4}, k(k+1)\right) \\
\frac{1}{4} & \text{otherwise} 
\end{cases}$$

**Region $\Omega_3$**

Proposition 1 and Lemma C.3 suggest the firm’s optimal profit to be

$$\Pi_3(q_t) = 2 - 2\sqrt{(1-q_t)^2 + c - k(1-q_t)} - q_t - k \Rightarrow \frac{\partial^2 \Pi_3(q_t)}{\partial q_t^2} = -\frac{k^2 - 4c}{2(1-q_t)(1-k-q_t)^{3/2}}.$$ 

First assume for now that $\frac{\partial^2 \Pi_{13}(q_t)}{\partial q_t^2} < 0$ and the first-order condition yields $q^*_t = 1 - \frac{k}{2} - \frac{\sqrt{12c - 3k^2}}{6}$ and $q^*_g = 1 - \frac{\sqrt{4c - k^2}}{3}$. Remind that $q^*_t$ and $q^*_g$ are feasible in region $\Omega_3$ only if $k \geq k_3(c, q^*_t, v, n)$ and $k \geq k_4(c, q^*_t, v, n)$ by Proposition 1, leading to $c \geq c_5$ and $c \leq c_7$ respectively. Next, we confirm that $\frac{\partial^2 \Pi_{13}(q_t)}{\partial q_t^2} < 0$ for $c_5 \leq c \leq c_7$.

First it can be shown that $c_5 \geq k^2$ so $q^*_t = 1 - \frac{k}{2} - \frac{\sqrt{12c - 3k^2}}{6}$ is feasible only for $c \geq k^2$, implying $k^2 - 4c < 0$. Second, $c + (1-q_t)(1-k-q_t) = \frac{1}{2}(4c - k^2) > 0$. Hence, we conclude that $\Pi_{13}(q_t)$ is concave in $q_t$ for $c_5 \leq c \leq c_7$. □

**Proof of Proposition 2.** The proof can mostly be obtained from direct inspection of the expressions of $q^*_t$ and $q^*_g$ in Table A.1 for regions $\Omega_0$, $\Omega_2$ and $\Omega_{12}$.

- For region $\Omega_1$,

$$\frac{\partial q^*_t}{\partial k} = -\frac{1}{2(1-v)} + \frac{k(1-2\sqrt{3-4v})}{2(3-4v)(1-v)\sqrt{4c(1-v)-k^2}} < 0 \Leftrightarrow k(1-2v) < \sqrt{(4c(1-v)-k^2)(3-4v)}$$

which apparently holds for $v \geq \frac{1}{2}$. When $v < \frac{1}{2}$, $k(1-2v) < \sqrt{(4c(1-v)-k^2)(3-4v)}$ implies $c < k^2(1-v)$. But it is straightforward to show that $\frac{k^2(1-v)}{4(1-v)} < k^2$ and also remember that $q^*_t$ is feasible in the region $\Omega_3$ only if $c \geq k^2$. Hence, we conclude that $c > \frac{k^2(1-v)}{4(1-v)}$ when $v < \frac{1}{2}$ and $q^*_t$ is feasible for region $\Omega_3$.

- For region $\Omega_3$, $q^*_t = \frac{6-3k-\sqrt{12c-3k^2}}{6}$, which is obviously decreasing in $c$. Straightforward calculation reveals

$$\frac{\partial q^*_t}{\partial k} = \frac{3k}{6} \sqrt{12c-3k^2} - 3 < \frac{1}{6} \left(\frac{3k}{\sqrt{12k^2-3k^2} - 3}\right) = -\frac{1}{3} < 0,$$

where the first inequality follows from the fact that $c > k^2$ in region $\Omega_3$. □
**Lemma E.1.** The firm enters the market if and only if \( c \leq (1 - q_t)q_t + (q_t - q_g)(p_g - k) \), in which case we can restrict the search for the optimal solution to the firm’s problem \((P)\) with additional constraint \( Q \leq \bar{Q} \) among the following regions:

1. If \( \bar{Q} \geq \frac{1 - n}{1 - q_t} \), then \( q_t \geq q_g \geq q_t - n/\bar{Q} \) and \( p_g = q_g + v(q_t - q_g) \).
2. If \( \bar{Q} \leq \frac{1 - n}{1 - q_t} \), then \( q_t \geq q_g \geq q_t - n/\bar{Q} \) and \( p_g = q_g + v(q_t - q_g) \).
3. If \( \bar{Q} \leq \frac{1 - n}{1 - q_t} \), \( q_g \leq q_t - n/\bar{Q} \) and \( p_g = q_g \).

**Proof.** As we discussed in §4, there are three possible cases:

1. \( Q_t = \bar{Q}(1 - q_t) \geq 1 - n \) (i.e., \( \bar{Q} \geq \frac{1 - n}{1 - q_t} \)) and \( Q_g = \bar{Q}(q_t - q_g) \leq n \) (i.e., \( q_g \geq q_t - \frac{n}{\bar{Q}} \)). These two inequalities imply that there is sufficient traditional products for all traditional consumers but the supply for green products is less than the size of green consumers. Thus, the firm sets \( p_g = q_g + v(q_t - q_g) \) so green consumers may purchase the traditional product.

2. \( Q_t = \bar{Q}(1 - q_t) \leq 1 - n \) (i.e., \( \bar{Q} \leq \frac{1 - n}{1 - q_t} \)) and \( Q_g = \bar{Q}(q_t - q_g) \leq n \) (i.e., \( q_g \geq q_t - \frac{n}{\bar{Q}} \)). These two inequalities imply that there are insufficient traditional products for all traditional (green) consumers and hence there is no spilling effect. The firm’s profit is \( \bar{Q}(1 - q_t)q_t + \bar{Q}(q_t - q_g)(p_g - k) - c\bar{Q} \), which is increasing in \( p_g \). Consequently, the firm chooses the premium price, \( p_g = q_g + v(q_t - q_g) \), as the optimal price for the green product.

3. \( Q_t = \bar{Q}(1 - q_t) \leq 1 - n \) (i.e., \( \bar{Q} \leq \frac{1 - n}{1 - q_t} \)) and \( Q_g = \bar{Q}(q_t - q_g) \geq n \) (i.e., \( q_g \leq q_t - \frac{n}{\bar{Q}} \)). These two inequalities imply that there are insufficient traditional products for all traditional consumers but there are sufficient green products for all green consumers. Thus, the firm sets \( p_g = q_g \) so traditional consumers may purchase the green products. \( \square \)

**Formulation of the sub-problems.** Note that when \( Q = \bar{Q} \), the firm’s objective function is

\[
\Pi(q_g, p_g) = \max_{q_g} \bar{Q}(1 - q_t)q_t + [p_g - k] \bar{Q}(q_t - q_g) - c\bar{Q},
\]

where \( p_g = q_g + v(q_t - q_g) \) in subproblem \( S_1 \) and \( S_2; \) \( p_g = q_g \) in subproblem \( S_3 \).

As the total supply of the products is no more than the total market demand, i.e., \( Q_t + Q_g = \bar{Q}(1 - q_t) \leq 1 \), we must have

\[
q_g \geq 1 - \frac{1}{\bar{Q}}.
\]

1. Part (1) of Lemma E.1 requires \( q_t \geq q_g \geq q_t - \frac{n}{Q} \) and \( \bar{Q} \geq \frac{1 - n}{1 - q_t} \) in subproblem 1. Since the latter inequality implies that \( 1 - \frac{1}{\bar{Q}} \geq q_t - \frac{n}{\bar{Q}} \), we thus can synthesize the former inequality and \((E.2)\) to \( q_t \geq q_g \geq \left(1 - \frac{1}{\bar{Q}}\right)^+ \), leading to the formulation of subproblem 1:

\[
\Pi_1 = \max_{q_g} \bar{Q}(1 - q_t)q_t + [q_g + v(q_t - q_g) - k] \bar{Q}(q_t - q_g) - c\bar{Q}
\]

subject to \( q_t \geq q_g \geq \left(1 - \frac{1}{\bar{Q}}\right)^+ \) and \( (1 - q_t)q_t + (q_t - q_g)[q_g + v(q_t - q_g) - k] \geq c \). \((E.3)\)

2. Part (2) of Lemma E.1 requires \( q_t \geq q_g \geq q_t - \frac{n}{Q} \) and \( \bar{Q} \leq \frac{1 - n}{1 - q_t} \) in subproblem 2. Again, since the latter inequality implies that \( q_t - \frac{n}{Q} \geq 1 - \frac{1}{Q} \). We can synthesize the former inequality and \((E.2)\) to \( q_t \geq q_g \geq \left(q_t - \frac{n}{Q}\right)^+ \), leading to the formulation of subproblem 2:

\[
\Pi_2 = \max_{q_g} \bar{Q}(1 - q_t)q_t + [q_g + v(q_t - q_g) - k] \bar{Q}(q_t - q_g) - c\bar{Q}
\]

subject to \( q_t \geq q_g \geq \left(q_t - \frac{n}{Q}\right)^+ \) and \( (1 - q_t)q_t + (q_t - q_g)[q_g + v(q_t - q_g) - k] \geq c \). \((E.4)\)
3. Part (3) of Lemma E.1 requires \( q_g \leq q_t - \frac{\alpha}{\tilde{Q}} \) and \( \tilde{Q} \leq \frac{1}{1-q_t} \) in subproblem 3. Again, since the latter inequality implies \( q_t - \frac{\alpha}{\tilde{Q}} \geq 1 - \frac{1}{\tilde{Q}} \), we can synthesize the former inequality and (E.2) to \( \left(1 - \frac{1}{\tilde{Q}}\right)^+ \leq q_g \leq \left(q_t - \frac{\alpha}{\tilde{Q}}\right)^+ \), leading to the formulation of subproblem 3:

\[
\Pi = \max_{q_g} \ Q(1-q_t)q_t + (q_g-k)\tilde{Q}(q_t-q_g) - c\tilde{Q}
\]
subject to \( \left(1 - \frac{1}{\tilde{Q}}\right)^+ \leq q_g \leq \left(q_t - \frac{\alpha}{\tilde{Q}}\right)^+ \) and \((1-q_t)q_t + (q_g-q_t)(q_g-k) \geq c. \)

(E.5)

**Lemma E.2 (Solution to subproblem 1).** The optimal solution to (E.3) is

\[
\text{for } q_t \geq n, \quad \tilde{q}_o = \begin{cases} q_t & \text{if } k \geq q_t, \\ \frac{k+q_t(1-2v)}{2(1-v)} & \text{if } \tilde{k}_0 \leq k \leq q_t, \\ q_t - \frac{\alpha}{\tilde{Q}} & \text{if } k \leq \tilde{k}_0, \end{cases}
\]

and for \( q_t < n, \quad \tilde{q}_o = \begin{cases} q_t & \text{if } k \geq q_t, \\ \frac{k+q_t(1-2v)}{2(1-v)} & \text{if } k \leq q_t, \end{cases}
\]

(E.6)

where \( \tilde{k}_0 \geq 0 \) implies \( \tilde{Q} \geq \frac{2(1-v)}{2(1-v)} \), and \( k \geq \tilde{k}_0 \) if and only if \( \tilde{Q} \geq Q < \frac{1}{1-q_t} \). Putting all these together yields (E.6). □

**Lemma E.3 (Solution to subproblem 2).** The optimal solution to (E.4) is

\[
\text{for } \begin{cases} q_t \geq n \text{ and } \frac{n}{q_t} \leq \tilde{Q} \leq \frac{1-n}{1-q_t}, \end{cases} \quad \tilde{q}_o = \begin{cases} q_t & \text{if } k \geq q_t, \\ \frac{k+q_t(1-2v)}{2(1-v)} & \text{if } \tilde{k}_1 \leq k \leq q_t, \\ q_t - \frac{\alpha}{\tilde{Q}} & \text{if } k \leq \tilde{k}_1, \end{cases}
\]

and for \( q_t \geq n, \tilde{Q} < \frac{n}{q_t} \) or \( q_t \leq n \), \n
\[
\tilde{q}_o = \begin{cases} q_t & \text{if } k \geq q_t, \\ \frac{k+q_t(1-2v)}{2(1-v)} & \text{if } k \leq q_t. \end{cases}
\]

(E.7)

where \( \tilde{k}_1 \geq 0 \) if \( v \leq \frac{1}{2} \), and \( k \geq \tilde{k}_1 \) if \( v > \frac{1}{2} \). Putting all these together yields (E.7). □

**Lemma E.4 (Solution to subproblem 3).** The optimal solution to (E.5) is

\[
\text{for } \begin{cases} q_t \geq n \text{ and } 1 \leq \tilde{Q} \leq \frac{1-n}{1-q_t}, \end{cases} \quad \tilde{q}_o = \begin{cases} q_t - \frac{\alpha}{\tilde{Q}} & \text{if } k \geq \tilde{k}_3, \\ \frac{k+q_t}{2(1-v)} & \text{if } \tilde{k}_3 \leq k \leq \tilde{k}_3, \\ q_t - \frac{\alpha}{\tilde{Q}} & \text{if } k \leq \tilde{k}_4, \end{cases}
\]

and for \( q_t \geq n, \tilde{Q} < 1 \), \n
\[
\tilde{q}_o = \begin{cases} q_t - \frac{\alpha}{\tilde{Q}} & \text{if } k \geq \tilde{k}_3, \\ \frac{k+q_t}{2(1-v)} & \text{if } k \leq \tilde{k}_4. \end{cases}
\]

(E.8)

where \( \tilde{k}_3 \geq 0 \) if \( v \leq \frac{1}{2} \) and \( \tilde{k}_4 \geq 0 \) if \( Q \geq \frac{2n}{q_t} \). Putting all these together yields (E.8). □
Proof. Again, $\frac{n}{q_t} \leq \frac{1-n}{1-q_t}$ implies $q_t \geq n$. Consequently, there are four possibilities on the constraint on $q_g$ in subproblem $S_3$: (a) When $q_t \geq n$ and $1 < \tilde{Q} < \frac{1-n}{1-q_t}$, we have $\tilde{Q} > 1 > \frac{n}{q_t}$, which imply that both sides of the first constraint in $S_3$ are positive, i.e., the first constraint simplifies to $1 - \frac{1}{\tilde{Q}} q_t \leq q_t - \frac{n}{q_t}$. (b) When $q_t \geq n$ and $\frac{n}{q_t} < \tilde{Q} < 1$, the first constraint simplifies to $0 \leq q_t \leq q_t - \frac{n}{q_t}$. (c) When $q_t \geq n$ and $\tilde{Q} \leq \frac{n}{q_t}$, the first constraint simplifies to $q_g = 0$. Also $p_g = q_g$ in this subproblem implies $p_g - k < 0$ in this case, indicating that this case is suboptimal. (d) When $q_t < n$ and $\tilde{Q} \leq \frac{n}{q_t}$, we have $\tilde{Q} \leq \frac{n}{q_t}$ because $\frac{1-n}{1-q_t} \leq \frac{n}{q_t}$. Then $q_t - \frac{n}{q_t} < 0$. The first constraint simplifies to $q_g = 0$.

The first order condition of the objective function in subproblem $S_4$ yields the interior optimal solution of $q_g^{FOC} = \frac{k + q_2}{2}$. We can verify that $q_g^{FOC} \leq q_t - \frac{n}{q_t}$ as long as $k \leq \tilde{k}_3 \equiv q_t - \frac{2n}{q_t}$. In addition, $q_g^{FOC} \geq 1 - \frac{1}{\tilde{Q}}$ as long as $k \geq \tilde{k}_3 \equiv 2 \left(1 - \frac{1}{\tilde{Q}}\right) - q_t$. In addition, $q_g^{FOC} \geq 0$ for all $k$. Putting all these together yields (E.8).

<table>
<thead>
<tr>
<th>Subproblem</th>
<th>Conditions</th>
<th>$q_g^{*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E.3): $\tilde{Q} \geq \frac{1-n}{1-q_t}; p_g^{<em>} = q_g + v(q_t - q_g^{</em>})$</td>
<td>$q_t \geq n$</td>
<td>$q_t$</td>
</tr>
<tr>
<td></td>
<td>$q_t &lt; n$</td>
<td>$2(1-v)$</td>
</tr>
<tr>
<td>(E.4): $\tilde{Q} \leq \frac{1-n}{1-q_t}; p_g^{<em>} = q_g + v(q_t - q_g^{</em>})$</td>
<td>$q_t \geq n$, $\frac{n}{q_t} \leq \tilde{Q} \leq \frac{1-n}{1-q_t}$</td>
<td>$q_t$</td>
</tr>
<tr>
<td></td>
<td>$(q_t \geq n$ and $\tilde{Q} \leq \frac{n}{q_t})$ or $q_t &lt; n$</td>
<td>$2(1-v)$</td>
</tr>
<tr>
<td>(E.5): $\tilde{Q} \leq \frac{1-n}{1-q_t}; p_g^{<em>} = q_g^{</em>}$</td>
<td>$q_t \geq n$, $1 &lt; \tilde{Q} \leq \frac{1-n}{1-q_t}$</td>
<td>$q_t$</td>
</tr>
<tr>
<td></td>
<td>$q_t &lt; n$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Proof of Proposition 3. First we consider $q_t \geq n$ and $v \leq \frac{1}{2}$. The former inequality implies $\frac{n}{q_t} \leq 1 \leq \frac{1-n}{1-q_t}$ which generates the following cases.

1. If $\tilde{Q} \leq \frac{n}{q_t}$, subproblem (E.3) is not feasible because it requires $\tilde{Q} \geq \frac{1-n}{1-q_t}$ but we have $\tilde{Q} \leq \frac{n}{q_t} \leq \frac{1-n}{1-q_t}$. In addition, subproblem (E.5) cannot be optimal because $q_g = 0$ in that case, leading to $p_g = q_g = 0 < k$, i.e., the firm lose money on every green product it produces. Therefore, only subproblem (E.4) is relevant. In addition, we have $q_g = \frac{k + q_t(1-2v)}{2(1-v)} > 0$ because $v \leq \frac{1}{2}$. Hence, the optimal solutions are: $p_g = q_g + v(q_t - q_g)$ and

$$q_g = \begin{cases} q_t + q_t(1-2v) & \text{if } k \geq q_t \\ \frac{k + q_t(1-2v)}{2(1-v)} & \text{if } k \leq q_t \end{cases}.$$
(2) If \( \bar{Q} \geq \frac{1-n}{1-q_t} \), then only subproblem (E.3) is relevant. In addition, we have \( q_y = \frac{k+q_t(1-2v)}{2(1-v)} > 0 \) because \( v \leq \frac{1}{2} \). Hence, the optimal solutions are: \( p_y = q_y + v(q_t - \bar{q}_y) \) and

\[
q_y = \begin{cases} 
q_t 
& \text{if } q_t \leq k \\
\frac{k+q_t(1-2v)}{2(1-v)} & \text{if } \tilde{k}_3 \leq k \leq q_t \\
1 - \frac{n}{q_t} & \text{if } k \leq \tilde{k}_3
\end{cases}
\]

(3) If \( \frac{n}{q_t} \leq \bar{Q} \leq \frac{1-n}{1-q_t} \), then both subproblem (E.4) and (E.5) are relevant. The optimal solutions of subproblem (E.4) and (E.5) are given in (E.7) and (E.8), respectively. Before we proceed with profit comparison, note that the revenue collected from selling the traditional products \( \bar{Q}(1-q_t)q_t \) and the cost of raw material \( c\bar{Q} \) are the same in both sub-problems. So we need to only compare the profits made from selling the green products. With a slight abuse of the notation, let \( \Pi(g_y, p_y) \) be the profits from selling the green product of quality \( q_y \) at price \( p_y \). First, we note that \( \tilde{k}_1 = q_t - \frac{2n(1-v)}{q_t} > q_t - \frac{2n}{q_t} = \tilde{k}_3 \). Then, we have the following two observation:

**Observation 1:** If \( \bar{q}_y \) takes the boundary solution \( q_t - \frac{n}{q_t} \) in both sub-problems, then subproblem (E.4) generates higher profit than (E.5) because the price for the green product \( \bar{p}_y \) is higher in (E.4) than in (E.5), i.e., \( \Pi(q_t - \frac{n}{q_t}, q_t - \frac{n(1-v)}{Q}) \geq \Pi(q_t - \frac{n}{q_t}, q_t - \frac{n}{q_t}) \).

**Observation 2:** Consequently, any solution in (E.4) dominates the boundary solution of \( \bar{q}_y = q_t - \frac{n}{q_t} \) in (E.5) because of the optimality of the optimal solution for subproblem (E.4).

On the other hand, although all three cases of (E.7) exist because \( \tilde{k}_i \geq 0 \) for all \( v \leq \frac{1}{2} \), some of the three cases of (E.8) may be irrelevant because \( \tilde{k}_3 \) or \( \tilde{k}_4 \) may be negative. First, we can verify that \( \frac{2n}{n+1} \leq \frac{2}{2-q_t} \leq \frac{1-n}{1-q_t} \) if \( q_t \geq \frac{2n}{n+1} \). Also notice that \( \frac{2n}{n+1} \geq n \) for any \( n \in [0, 1] \). Therefore, if \( q_t \geq \frac{2n}{n+1} \) and \( \frac{2n}{n+1} \leq \bar{Q} \leq \frac{1-n}{1-q_t} \), both \( \tilde{k}_3 \geq 0 \) and \( \tilde{k}_4 \geq 0 \) so all three cases in E.8 are relevant. If \( q_t \geq \frac{2n}{n+1} \) and \( \frac{2n}{n+1} \leq \bar{Q} \leq \frac{2}{2-q_t} \), then \( \tilde{k}_4 \leq 0 \) so the bottom case is irrelevant. If \( q_t \geq \frac{2n}{n+1} \) and \( \frac{n}{q_t} \leq \bar{Q} \leq \frac{2n}{n+1} \), we all know \( \frac{n}{q_t} \leq \frac{1-n}{1-q_t} \leq \frac{2n}{n+1} \). This implies that for any \( \frac{n}{q_t} \leq \bar{Q} \leq \frac{1-n}{1-q_t} \), we have \( \tilde{k}_4 \leq \tilde{k}_3 \leq 0 \) so the bottom two cases are irrelevant. Consequently, comparison of the firm’s profit between subproblem (E.4) and (E.5) comes down to three cases: (a) when \( q_t \geq \frac{2n}{n+1} \) and \( \frac{2n}{n+1} \leq \bar{Q} \leq \frac{1-n}{1-q_t} \); (b) when \( q_t \geq \frac{2n}{n+1} \) and \( \frac{2n}{n+1} \leq \bar{Q} \leq \frac{2}{2-q_t} \); and (c) when \( q_t \geq \frac{2n}{n+1} \) and \( \frac{n}{q_t} \leq \bar{Q} \leq \frac{2n}{n+1} \) or when \( n \leq q_t \leq \frac{2n}{n+1} \) and \( \frac{n}{q_t} \leq \bar{Q} \leq \frac{1-n}{1-q_t} \).

(3a) When \( q_t \geq \frac{2n}{n+1} \) and \( \frac{2n}{n+1} \leq \bar{Q} \leq \frac{1-n}{1-q_t} \), the optimal \( \bar{q}_y \) in (E.4) is (E.7) and the optimal \( q_y \) in (E.5) is (E.8) with all three cases being relevant. It is straightforward that for \( k \geq \tilde{k}_3 \), the optimal \( q_y \) is given by

\[
q_y = \begin{cases} 
q_t + \frac{q_t(1-2v)}{2(1-v)} & \text{if } k \geq q_t \\
q_t & \text{if } \tilde{k}_1 \leq k \leq q_t \\
\frac{n}{q_t} & \text{if } \tilde{k}_3 \leq k \leq \tilde{k}_1
\end{cases}
\]

For \( \tilde{k}_4 \leq k \leq \tilde{k}_3 \), we need to compare \( \Pi(\bar{Q}, q_t - \frac{n}{q_t}, \bar{Q} - \frac{n(1-v)}{Q}) \) with \( \Pi(\bar{Q}, \frac{k+q_t(1-2v)}{2(1-v)}, \frac{k+q_t(1-2v)}{2(1-v)} - k) \) using the boundary solution of (E.4), with \( \Pi(\bar{Q}, q_t - \frac{n}{q_t}, \bar{Q} - \frac{n(1-v)}{Q}) \) and \( \Pi(\tilde{k}_2, q_t - \frac{n}{q_t}, \frac{2n}{n+1} - k) \), the profit obtained by using the interior solution of (E.5). Comparing these two profits directly, we know that \( \frac{1}{4}(q_t - k)^2 \bar{Q} \geq n \left[ q_t - \frac{n(1-v)}{Q} - k \right] \) when \( k \leq \tilde{k}_2 \equiv q_t - \frac{2n(1+\sqrt{\frac{v}{1-q_t}})}{Q} \). It is easy to verify that \( \tilde{k}_2 < q_t - \frac{2n}{Q} = \tilde{k}_3 \) for any \( v \geq 0 \). In addition, \( \tilde{k}_2 = q_t - \frac{2n(1+\sqrt{\frac{v}{1-q_t}})}{Q} \geq 2 \left( 1 - \frac{1}{4} \right) - q_t = \tilde{k}_4 \) for \( \bar{Q} \leq \frac{1-n(1+\sqrt{\frac{v}{1-q_t}})}{1-q_t} \). Therefore, \( \bar{Q} \geq \frac{1-n(1+\sqrt{\frac{v}{1-q_t}})}{1-q_t} \) implies \( \tilde{k}_2 \leq \tilde{k}_4 \). Thus, for any
\( \hat{k}_4 \leq k \leq \hat{k}_3 \), the profit from the boundary solution of (E.4) is larger than the profit from the interior solution of (E.5) because \( k \geq \hat{k}_2 \). Hence, \( q_g = q_1 - \frac{n}{Q} \). If \( Q \leq \frac{1-n(1+\sqrt{t})}{1-q_1} \), we have \( \hat{k}_4 \leq \hat{k}_2 \leq \hat{k}_3 \). Then, if \( \hat{k}_2 \leq k \leq \hat{k}_3 \), the boundary solution in (E.4) generates higher profit than the interior solution in (E.5) so \( q_g = q_1 - \frac{n}{Q} \); and if \( \hat{k}_4 \leq k \leq \hat{k}_2 \), the boundary solution in (E.4) generates less profit than the interior solution in (E.5) so \( q_g = \frac{k+q_2}{2} \).

For \( 0 \leq k \leq \hat{k}_4 \), we need to compare \( \Pi\left(q_1 - \frac{n}{Q}, q_1 - \frac{n(1-v)}{Q}\right) = n\left[q_1 - \frac{n(1-v)}{Q} - k\right]\), the profit obtained by using the boundary solution of (E.4), with \( \Pi\left(1 - \frac{q}{Q}, 1 - \frac{q}{Q}\right) = Q\left(1 - \frac{q}{Q} - k\right)\left(q_1 - 1 + \frac{q}{Q}\right)\), the profit obtained by using the boundary solution of (E.5). Both profits are decreasing in \( k \). At \( k = \hat{k}_4 = 2\left(1 - \frac{1}{Q}\right) - q_t \), the former profit simplifies to \( n\left[2q_1 - \frac{n(1-v)}{Q} - 2\left(1 - \frac{q}{Q}\right)\right] \) and the latter profit simplifies to \( Q\left(q_1 - 1 + \frac{q}{Q}\right)^2 \). We can verify that for \( \hat{Q} \geq \frac{1-n(1+\sqrt{t})}{1-q_1} \), we have \( Q\left(q_1 - 1 + \frac{q}{Q}\right)^2 \geq n\left[2q_1 - \frac{n(1-v)}{Q} - 2\left(1 - \frac{q}{Q}\right)\right] \), which leads to the profit generated by the boundary solution in (E.5) is bigger than that by the boundary solution in (E.4). When \( \hat{Q} \geq \frac{1-n(1+\sqrt{t})}{1-q_1} \), we compare \( \Pi\left(q_1 - \frac{n}{Q}, q_1 - \frac{n(1-v)}{Q}\right) \) to \( \Pi\left(1 - \frac{q}{Q}, 1 - \frac{q}{Q}\right) \) and find that the profit in (E.5) is bigger than the one in (E.4) as long as \( k \leq \frac{\hat{Q}(1-Q)\mu - (1-Q)^2 - \bar{q}_n \bar{Q} + n^2(1-v)}{Q(1-n-Q+\bar{Q}n)} \). Denote \( \hat{k}_5 = \min\left\{\frac{\hat{Q}(1-Q)\mu - (1-Q)^2 - \bar{q}_n \bar{Q} + n^2(1-v)}{Q(1-n-Q+\bar{Q}n)}, \hat{k}_4 \right\} \). We can summarize the optimal solutions as the following:

(i) When \( q_t \geq \frac{2n}{1+n} \) and \( \frac{2}{2-q_t} \leq Q \leq \frac{1-n(1+\sqrt{t})}{1-q_1} \), we have:

\[
(p_g^*, p_g^*) = \begin{cases} 
(q_t, N/A) & \text{if } k \geq q_t \\
\left(\frac{k+q_1(1-2q)}{2(1-q)}, \frac{k+q_2}{2}\right) & \text{if } k_1 \leq k \leq q_t \\
\left(q_1 - \frac{n}{Q}, q_1 - \frac{n(1-v)}{Q}\right) & \text{if } k_2 \leq k \leq \hat{k}_1 \\
\left(1 - \frac{q}{Q}, 1 - \frac{q}{Q}\right) & \text{if } k \leq \hat{k}_4 
\end{cases}
\]

(ii) When \( q_t \geq \frac{2n}{1+n} \) and \( \frac{1-n(1+\sqrt{t})}{1-q_1} \leq \hat{Q} \leq \frac{1-n}{1-q_1} \), we have:

\[
(p_g^*, p_g^*) = \begin{cases} 
(q_t, N/A) & \text{if } k \geq q_t \\
\left(\frac{k+q_1(1-2q)}{2(1-q)}, \frac{k+q_2}{2}\right) & \text{if } k_1 \leq k \leq q_t \\
\left(q_1 - \frac{n}{Q}, q_1 - \frac{n(1-v)}{Q}\right) & \text{if } k_5 \leq k \leq \hat{k}_1 \\
\left(1 - \frac{q}{Q}, 1 - \frac{q}{Q}\right) & \text{if } k \leq \hat{k}_5 
\end{cases}
\]

(3b) When \( q_t \geq \frac{2n}{1+n} \), \( \frac{2n}{q_t} \leq \hat{Q} \leq \frac{2}{2-q_t} \), the optimal \( q_g \) in (E.4) is (E.7) and the optimal \( q_g \) in (E.5) is (E.8) with top two cases. The analysis is similar to those in (3c) except that \( q_g = 1 - \frac{\bar{q}}{Q} \) is never an optimal solution to (E.5). Thus, the optimal solutions are:

\[
(p_g^*, p_g^*) = \begin{cases} 
(q_t, N/A) & \text{if } q_t \leq k \\
\left(\frac{k+q_1(1-2q)}{2(1-q)}, \frac{k+q_2}{2}\right) & \text{if } k_1 \leq k \leq q_t \\
\left(q_1 - \frac{n}{Q}, q_1 - \frac{n(1-v)}{Q}\right) & \text{if } k_2 \leq k \leq \hat{k}_1 \\
\left(\frac{k+q_1(1-2q)}{2(1-q)}, \frac{k+q_2}{2}\right) & \text{if } k \leq \hat{k}_2 
\end{cases}
\]

(3c) When \( q_t \geq \frac{2n}{1+n} \) and \( \frac{n}{q_t} \leq \hat{Q} \leq \frac{2n}{q_t} \) or when \( n \leq q_t \leq \frac{2n}{1+n} \) and \( \frac{n}{q_t} \leq \hat{Q} \leq \frac{1-n}{1-q_1} \), the optimal \( q_g \) in (E.5) is just the boundary solution \( q_t - \frac{n}{Q} \), which is dominated by any solutions in (E.4). Hence, the optimal solutions for the firm are \( p_g = q_g + v(q_t - q_g) \) and
When \( q_t < n \), Table E.1 shows that sub-problems (E.3) and (E.4) have the same solution. In addition, subproblem (E.5) yields \( q_g = 0 \) and it’s profit is lower than that of sub-problems (E.3) and (E.4). Therefore when \( q_t < n \) we have \( q_g = q_t \) for \( k \geq q_t \) and \( q_g = \frac{k + q_t (1 - 2v)}{2(1-v)} \) otherwise. When \( v > \frac{1}{2} \), we have \( \tilde{k}_1 < 0 \) so the case \( q_g = qt - n\bar{Q} \) for subproblem (E.4) becomes irrelevant but otherwise everything else will be the same. \( \square \)